Why Do Large Investors Disclose Their Information?*

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Abstract

Large investors often advertise private information at private talks or in the media. To analyse the incentives for information disclosure, I develop a two-period Kyle (1985) type model in which an informed short-horizon investor strategically discloses private information to enhance price efficiency. I show that information disclosure is optimal when the scope of private information is large and when the large investor has a high reputation. Short investment horizons induce information disclosure among investors and are beneficial for price efficiency. However, strategic information disclosure reduces trading before disclosure and harms price discovery.

Keywords: Information Disclosure, Price Discovery, Asymmetric Information, Market Microstructure

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Large investors, such as hedge fund managers, often advertise their trading ideas at private talks or in the media. Academic researchers have documented that this is a systematic phenomenon. In their recent empirical work, Ljungqvist and Qian (2016) examine available public reports from some hedge funds and individual investors about shorting 124 overpriced companies. They found that these investors managed to correct the mispricing by encouraging long-term investors to sell their positions on the target companies. Swem (2017) also discovered that hedge funds usually acquire information, trade and then strategically disclose their information to mutual funds via analysts, expecting mutual funds to trade on their information and, thus, accelerate the incorporation of information into the asset price.

The above evidence leads to the following questions: Why do investors spread their information freely, considering that information production is very costly? Why do these investors not trade and profit from the valuable information, instead of disclosing it? Furthermore, does this behavior improve or harm price efficiency and trading intensity?

To answer these questions, I developed a two-period model based on Kyle (1985) in which endogenous information disclosure arises due to the short horizon of the informed investor. In the model, there is one risky asset, whose payoff is only known by the informed investor. Trading takes place in both periods and the payoff of the asset is exogenously realized at the end of the second period. The informed investor trades the asset in the first period and liquidates the position in the second period. Before liquidation, the informed investor decides to disclose the private information to other uninformed investors or to liquidate the position silently. The informed investor trades rationally in the first period by taking the information disclosure into consideration.

Both silent liquidation and endogenous information disclosure lead to early information revelation and, thus, improve price informativeness and accelerate the speed of mispricing correction. Silent liquidation is achieved through the aggressive trading of the informed investor himself or herself, and endogenous information disclosure is achieved through the participation of new investors who receive the information. The information disclosure decision depends on the price realization in the first period and the trading intensity of the new investors. The informed investor tends to disclose the information when the number of followers is large. Information disclosure is most likely to occur when the informed investor has a higher reputation. With information disclosure, the informed investor reduces the trading intensity.

The short horizon of informed investors is natural in the model. In contrast to long-term investors for whom the liquidation price is the known fundamental value of the asset, the short-term investor faces uncertainty about the liquidation price. Thus, the horizon limit results in different trading behaviors. Long-term investors trade cautiously to hide their private information and alleviate price impacts. However, short-term investors are better off hiding information and eliminating price impacts when entering the market and disclosing information and creating price impacts when leaving the market. The two opposite targets cannot be achieved by purely adjusting the trading intensity because the informed investor's trading volumes are the same at the two points of time. Therefore, endogenous information disclosure arises when the informed investor liquidates his or her position. The disclosure attracts new investors and additional capital flows into the market and pushes prices closer to their fundamental values.

Silently liquidating the position could be optimal in this model because the informed investor has price impacts. The informed trader incurs no financial constraint and is able to make a large order that moves the price. In case the number of followers is small or the financial capital brought by the followers is not large enough, the informed trader is better off taking a large position and signal the information to the market maker. However, there is always a risk that the information might be revealed early.

One question is under which conditions does information disclosure happen? I find that the investor discloses the information when the trading volume of the new investors is at least two times the trading volume of the informed investor. The market maker gets the information through the total number of orders she observes: the presence of large orders indicates that there is much information. Without new investors, the total order in the second period is only the liquidation plus the noise demand. With new investors, half of the demand from new investors is unobservable to the market maker because it offsets the liquidation of the informed investor. The remaining half, mixed with the order from noise traders, is observed by the market maker. That is why the disclosure decision is related to two times the trading volume of new investors.

Another interesting question is about the impact of information disclosure on price discovery and market efficiency. It is evident that information disclosure improves price efficiency after disclosure because additional informed investors participate in the market after information disclosure occurs, and thus, the prices are highly informative. Related literature, such as that of Bommel (2003), Kovbasyuk and Pagano (2015), Ljungqvist and Qian (2016) and Schmidt (2017) also confirm this result. However, they focus on small or competitive investors who incur financial constraints and do not have any price impacts. I focus on large investors and find that information disclosure reduces trading intensity and harms price discovery before disclosure. In the presence of information disclosure, an informed investor can "borrow" financial firepower from his or her followers in the second period. Thus, he or she only focuses on hiding the information in the first period and, therefore, trades less intensively. In the model I show that with the option to disclose information, the short-term informed trader trades even less than a long-term trader.

The paper generates important policy implications in terms of information sharing among investors. Previous studies promote information disclosure because it eliminates the limit on arbitrage and encourages the participation of short-term investors. I complement these results by showing that the impact of information disclosure by large investors is different than that of others. I find that strategic information disclosure reduces the trading intensity before disclosure and harms price discovery. Information disclosure is beneficial for the informed investor but detrimental to the noise traders because the expected loss of the noise traders increases with information disclosure.

Related Literature

Other papers on the incentives of information disclosure include the theoretical works of Bommel (2003), Kovbasyuk and Pagano (2015), and Schmidt (2017) and the empirical paper of Ljungqvist and Qian (2016). The informed investors (rumourmongers or speculators) in these papers are small and competitive, and face capital constraints. The orders from the investors are negligible and do not have price impacts. In contrast, I investigate the behavior of large investors, who usually have large demand and cause price adjustments. Information disclosure is a strategic decision that takes price impacts into account. Moreover, I show that an informed investor strategically reduces demand before disclosure to hide information and eliminate price impacts.

My paper is also related to two other strands of literature. One is about information sales and exchanges. Admati and Pfleiderer (1986) document that, for a monopolistic information owner, it is optimal to add noise to the information when it is sold and that it is more profitable to sell different signals to different traders than just one signal to all traders. Admati and Pfleiderer (1988) discussed whether an informed owner wants to sell information or alternatively trade strategically on the basis of the information. Fishman and Hagerty

(1995) explain that selling information is a way for informed traders to aggressively commit to trades. The authors also point out that a risk-averse trader is better off selling information than just trading on it. Information disclosure is similar to information selling with zero price. However, I found that zero-price information does not exist in the equilibrium results of the above papers. Because the informed investor has to compensate his or her losses from competing with new investors using the profit from information sales, the price can not be zero. Studies on information exchange, such as that of Stein (2008), shows that the information is transmitted among investors. The complementarity of information structures makes it optimal to exchange information with other investors. Because the informed investor does not necessarily expect information feedback from the receivers, the information transmission mechanism can not explain the disclosure.

The second part is about the limit on arbitrage. In the survey of Gromb and Vayanos (2010), the authors summarize the costs incurred by arbitrageurs, including fundamental and non-fundamental risks, short-selling costs, and leverage, margin, and capital constraints. In particular, De Long et al. (1990) documents noise trader risk. Shleifer and Vishny (1997) discuss performance-based arbitrage, whereby investors face capital constraints. Abreu and Brunnermeier (2002, 2003) introduce the synchronization risk: arbitragers' uncertainty about when other arbitrageurs will start exploiting a common arbitrage opportunity causes delayed arbitrage with holding costs. My paper complements the literature by pointing out a way of eliminating the limit on arbitrage.

The paper is also related to the mandatory disclosure literature, such as that of Steven Huddart (2001) and Yang and Zhu (2017). These researcher also use the two-period Kyle (1985) trading mechanism in their papers. In contrast, I endogenize the information disclosure decision and derive a sub-game perfect equilibrium. Steven Huddart (2001) shows that mandatory information disclosure is beneficial to price discovery and market efficiency while

Yang and Zhu (2017) conclude that the presence of order flow traders harms price discovery in the first period and its impact on market liquidity is mixed. Yang and Zhu (2017) find that the price discovery is harmed because informed investors add noise to demand, but I show that the price discovery is harmed because the informed investor reduces the trading intensity.

I proceed as follows. In Section 1, I describe the model. In Section 2, I solve the benchmark case in which the short-term trader does not disclose the information. Then I introduce the information disclosure in Section 3 and derive the sub-game perfect equilibrium. I discuss the implications of information disclosure in Section 4. Section 5 concludes the paper.

1 The Model

In the model, one single risky asset is traded by four market participants: one informed trader, one competitive market maker, noise traders and followers. There are three time periods $t \in \{1, 2, 3\}$, the risky asset is traded in periods 1 and 2 before it pays off at the third period. The asset's payoff θ is either high (θ_H) or low (θ_L) with probability β_0 and $1-\beta_0$. The initial price of the asset equals to the expectation of the payoff, that is $p_0 = \beta_0\theta_H + (1-\beta_0)\theta_L$. For simplicity, I assume that $\beta_0 = \frac{1}{2}$.

At $t = 1$, the informed trader arrives in the market with probability $\mu \in [0, 1]$, whether the informed trader is present or not is unknown to anyone else. The informed trader is risk neutral and has short horizon, that is, the informed trader trades in the first period and liquidates the position in the second period. In addition, the short horizon of the informed trader is common knowledge. The informed trader knows the payoff of the risky asset and trades strategically with the information. I follow Grossman and Stiglitz (1982) by introducing noise traders in the market, who do not have any information and trade randomly. The existence of noise traders ensures that the information of informed trader will not be fully reflected in price and the informed trader does not take all the surplus. The demand from noise traders in each period are independently uniformly distributed[1], $u_t \sim U(-1, 1)$.

As in Kyle (1985), the price of the risky asset in each period is set by a competitive market maker. The market maker only observes the total order flow y_t and sets the price p_t to clear the market:

$$p_1 = E[\theta|y_1], \quad p_2 = E[\theta|y_1, y_2].$$

The informed trader has to close the position in period 2, while the payoff is realized in period 3. The informed trader thus faces uncertainty about the liquidation price. In order to eliminate the price uncertainty, the informed trader may disclose the private information to followers. The followers are uninformed at period 1 and do not participate in the market. They only get informed and start trading if the informed trader discloses the information at period 2. The number of followers m is observable to the informed trader at period 1.

Credibility is essential in the information revelation, in this model, I simply restrict that the informed trader only reveals true information. The assumption is reasonable since the objective is large investors in the financial market, for example, fund managers, who concerns a lot of the reputation. Once the large investor is confirmed as spreading false rumours and manipulating the market, her reputation will be dramatically impaired and she may lose the job or face large investment withdraw. Moreover, a recent paper by Schmidt (2017) also states that short-term rumourmonger prefers to share her information truthfully.

[1]Same assumption in Schmidt (2017)

2 Benchmark: No information disclosure

In this section, I derive the equilibrium of the trading game without information disclosure. In particular, I characterize the pricing equations and compute the optimal strategy of the informed trader. The equilibrium is solved using forward-backward induction. I first characterize the price function in the first period. Then I derive the price equation in the second period. Finally, I compute the optimal demand of the informed trader.

2.1 Price in First Period

In the first period, if the informed trader appears, she buys x_{1l} if she learns that $\theta = \theta_H$ and sells x_{1s} if she learns that $\theta = \theta_L$. The noise traders always appear in the market and trade u_1 in the first period. The total order flow in the first period is:

$$y_1 = \begin{cases} x_{1l} + u_1 & \text{if} \quad \theta = \theta_H \quad \text{and informed trader appears} \\ u_1 & \text{if} \quad \text{informed trader does not appear} \\ -x_{1s} + u_1 & \text{if} \quad \theta = \theta_L \quad \text{and informed trader appears} \end{cases}$$

The market maker is uninformed, who tries to extrapolate information from the order flow she observes. Due to the competitiveness, the market maker sets the price equal to the expected value of the risky asset conditional on the total order flow y_1. Since the demand from the noise traders are uniformly distributed and bounded on $[-1, 1]$, the total order in the first period is distributed on $[-x_{1s} - 1, x_{1l} + 1]$. Depending on the value of x_{1l} and x_{1s}, the price equations are as follows:

(1) $0 \leq x_{1l} < 1$ and $0 \leq x_{1s} < 1$,

$$p_1 = E[\theta|y_1] = \begin{cases} \theta_H & \text{if} \quad 1 < y_1 \leq 1 + x_{1l} \\ p_M & \text{if} \quad 1 - x_{1s} < y_1 \leq 1 \\ p_0 & \text{if} \quad x_{1l} - 1 \leq y_1 \leq 1 - x_{1s} \\ \tilde{p}_M & \text{if} \quad -1 \leq y_1 < x_{1l} - 1 \\ \theta_L & \text{if} -x_{1s} - 1 \leq y_1 < -1 \end{cases} \tag{1}$$

(2) $1 \leq x_{1l} < 2$ and $1 \leq x_{1s} < 2$,

$$p_1 = E[\theta|y_1] = \begin{cases} \theta_H & \text{if} \quad 1 < y_1 \leq 1 + x_{1l} \\ p_M & \text{if} \quad x_{1l} - 1 < y_1 \leq 1 \\ p_0 & \text{if} \quad 1 - x_{1s} \leq y_1 \leq x_{1l} - 1 \\ \tilde{p}_M & \text{if} \quad -1 \leq y_1 < 1 - x_{1s} \\ \theta_L & \text{if} \quad -x_{1s} - 1 \leq y_1 < -1 \end{cases} \tag{2}$$

(3) $x_{1l} \geq 2$ and $x_{1s} \geq 2$,

$$p_1 = E[\theta|y_1] = \begin{cases} \theta_H & \text{if} \quad x_{1l} - 1 \leq y_1 \leq x_{1l} + 1 \\ p_0 & \text{if} \quad -1 \leq y_1 \leq 1 \\ \theta_L & \text{if} \quad -x_{1s} - 1 \leq y_1 \leq 1 - x_{1s} \end{cases}$$

where $p_M \equiv \beta_1\theta_H + (1-\beta_1)\theta_L$ and $\tilde{p}_M \equiv (1-\beta_1)\theta_H + \beta_1\theta_L$. $\beta_1 = \frac{\beta_0}{\beta_0\mu+1-\mu} = \frac{1}{2-\mu}$, which is the updated belief of market maker conditional on observing a total order y_1 in-between $1 - x_{1s}$ and 1 in case (1) or between $x_{1l} - 1$ and 1 in case (2).

Different from Kyle (1985), the price equation is piecewise throughout the entire domain, that is due to the binomial distribution of the payoff θ. The price is closer to the true payoff when the total order size $|y_1|$ is large. For instance, when y_1 is in-between 1 and $1 + x_{1l}$ in case (1) and (2), or between $x_{1l} - 1$ and $x_{1l} + 1$ in case (3), the market maker can infer the existence of informed trader since the order from noise trader can not be larger than 1. From the positive order, the market maker can further deduce that the payoff is high and set the price as θ_H. As shown by case (3), when the informed trader trades very aggressively, the private information is fully revealed through trading in the first period, and the informed trader gets zero profit. Therefore, very aggressive order, that is $x_{1l} \geq 2$ and $x_{1s} \geq 2$ are not optimal and will not considered thereafter.

When the total order size $|y_1|$ is moderate, the market maker can not differentiate whether the order comes from purely noise trader, or from both noise trader and informed trader. But conditional on the existence of informed trader, the market maker can deduce the true payoff from the order direction, so she makes an adjustment on the price. For example, when y_1 is between $1 - x_{1s}$ and 1, the market maker deduces that the order comes from both informed trader and noise trader with probability $\beta_0\mu$, the order comes from purely noise trader with probability $1 - \mu$. So the market maker updates the belief of high payoff from β_0 to $\beta_1 = \frac{\beta_0}{\beta_0\mu+1-\mu}$, and sets the price as p_M. Similarly, when the market maker observes a negative order between -1 and $x_{1l} - 1$, her belief of low payoff is adjusted to $\frac{1-\beta_0}{1-\beta_0+1-\mu} = \beta_1$, and sets the price as $\tilde{p}_M$.

At last, when the order size $|y_1|$ is very small, which is the intersection of $x_{1l} + u_1$, u_1 and $-x_{1s} + u_1$, the market maker can not extrapolate any information from the total order, so she keeps the price as p_0.

Next, I characterize the prices in the second period.

2.2 Price in Second Period: no information disclosure

In the second period, the total order includes the demand from noise traders u_2, and, in presence of informed trader, the liquidation from the informed trader:

$$y_2 = \begin{cases} -x_{1l} + u_2, & \text{if} \quad y_1 = x_{1l} + u_1, \\ u_2, & \text{if} \quad y_1 = u_1, \\ x_{1s} + u_2, & \text{if} \quad y_1 = -x_{1s} + u_1. \end{cases}$$

The market maker sets the price equal to the expected value of risky asset conditional on the current order flow y_2 and historical order y_1:

$$p_2 = E[\theta | y_1, y_2] = E[\theta | p_1, y_2].$$

Since p_1 contains the same information as y_1, the market maker thus adjusts the price p_2 based on p_1 and y_2.

Conditional on high payoff θ_H and the appearance of informed trader, there are three possible outcomes of price p_1: θ_H, p_M and p_0. Firstly, It is obvious that once the information is fully revealed in the first period, that is $p_1 = \theta_H$, the price $p_2 = p_1 = \theta_H$. Then I show the price equations when (1) $p_1 = p_M$; (2) $p_1 = p_0$ in below.

(1) When $p_1 = p_M$

$$p_2 = \begin{cases} \theta_H, & \text{if} \quad -1 - x_{1l} \leq y_2 \leq -1, \\ p_M, & \text{if} \quad -1 < y_2 \leq 1 - x_{1l}, \\ p_0, & \text{if} \quad 1 - x_{1l} < y_2 \leq 1. \end{cases} \tag{3}$$

(2) When $p_1 = p_0$:

$$p_2 = \begin{cases} \theta_H, & \text{if} \quad -1 - x_{1l} \leq y_2 < -1, \\ p_M, & \text{if} \quad -1 \leq y_2 < x_{1s} - 1, \\ p_0, & \text{if} \quad x_{1s} - 1 \leq y_2 \leq 1 - x_{1l}, \\ \tilde{p}_M, & \text{if} \quad 1 - x_{1l} < y_2 \leq 1, \\ \theta_L, & \text{if} \quad 1 < y_2 \leq x_{1s} + 1. \end{cases} \tag{4}$$

In contrast to the pricing rule in the first period, a large positive total order in the second period implies low payoff and a large negative total order implies high payoff That is due to the liquidation of the informed trader. Since the short-term horizon of informed trader is common knowledge, the market maker realizes that in the presence of the informed trader, the order must be negative in the first period if she observes a large positive order in the second period, and then she adjusts her belief downwards, vice versa.

If the trading in the first period partially reveals the information and $p_1 = p_M$, the market maker ruled out the possibility that $y_1 = -x_{1s} + u_1$. Then in the second period, the total order is either $y_2 = -x_{1l} + u_2$ or $y_2 = u_2$, and the price p_2 is irrelevant with x_{1s}, as shown by equation (3). If the trading in the first period does not reveal any information, that is when $p_1 = p_0$, the price p_2 depends on both x_{1l} and x_{1s}, as shown by equation (4).

2.3 The demand of informed trader

Given the price equations in each period, I can compute the expected profit of the informed trader and then derive the optimal demand. Since the probability of high payoff equals to the probability of low payoff, the equilibrium is symmetric. So I only study the case when the payoff is high. For comparison, I calculate the optimal demands of informed trader with short-term horizon and that with long-term horizon. Furthermore, I impose one tie-breaking rule: when two orders result in the same expected profit, the informed trader strictly prefers the smaller order.

At first, the expected profit of the informed trader with long-term horizon is given by

$$\begin{aligned} E[\pi_1^L|\theta=\theta_H] &= E[x_{1l}(\theta_H - p_1)|\theta=\theta_H] \\ &= \begin{cases} x_{1l}\left[(\theta_H - p_M)\frac{x_{1s}}{2} + (\theta_H - p_0)\frac{2-x_{1l}-x_{1s}}{2}\right], & 0 \le x_{1l} < 1, 0 \le x_{1s} < 1, \\ x_{1l}\left[(\theta_H - p_M)\frac{2-x_{1l}}{2}\right], & 1 \le x_{1l} < 2, 1 \le x_{1s} < 2. \end{cases} \end{aligned}$$

The first order condition gives the optimal strategies in each subdomain as

$$x_{1l}^{L*} = \begin{cases} 1 - \frac{\beta_1 - \beta_0}{2(1-\beta_0)} x_{1s}, & 0 \le x_{1l} < 1, 0 \le x_{1s} < 1, \\ 1, & 1 \le x_{1l} < 2, 1 \le x_{1s} < 2. \end{cases}$$

Since $\beta_0 = \frac{1}{2}$, the informed trader plays symmetric strategy when the true payoff is high and low, that is $x_{1l} = x_{1s}$, which yields the optimal demand of informed trader with long-term horizon, denoted by x_L as

$$x_L = x_{1l}^{L*} = x_{1s}^{L*} = \frac{2(1-\beta_0)}{2+\beta_1 - 3\beta_0} = \frac{1}{\beta_1 + \frac{1}{2}}.$$

Next, when the informed trader has short-term horizon, the expected profit is

$$\begin{aligned} E[\pi_1^S|\theta=\theta_H] &= E[x_{1l}(p_2 - p_1)|\theta=\theta_H] \\ &= \begin{cases} x_{1l}\left[(\theta_H - p_M)\frac{x_{1s}}{2}\frac{x_{1l}}{2} + (\theta_H - p_0)\frac{2-x_{1l}-x_{1s}}{2}\frac{x_{1l}}{2} + (p_M - p_0)\frac{2-x_{1l}-x_{1s}}{2}\frac{x_{1s}}{2}\right], & 0 \le x_{1l} < 1, \\ & 0 \le x_{1s} < 1, \\ x_{1l}\left[(\theta_H - p_M)\frac{2-x_{1l}}{2}\frac{x_{1l}}{2}\right], & 1 \le x_{1l} < 2. \end{cases} \end{aligned}$$

Solving the first order condition gives the optimal strategies in each subdomain as

$$x_{1l}^{S*} = \begin{cases} \frac{2(1-\beta_0)-2(\beta_1-\beta_0)x_{1s}-\sqrt{(\beta_1-\beta_0)(4\beta_1-\beta_0-3)x_{1s}^2-2(1-\beta_0)(\beta_1-\beta_0)x_{1s}+4(1-\beta_0)^2}}{3(1-\beta_0)}, & 0 \le x_{1l} < 1, 0 \le x_{1s} < 1, \\ \frac{4}{3}, & 1 \le x_{1l} < 2. \end{cases}$$

Taking $x_{1l}^* = x_{1s}^*$ and then comparing the expected profit in each subdomain by substituting the x_{1l}^*, x_{1s}^* into the profit function, gives the optimal demand as

$$x_S = x_{1l}^{S*} = x_{1s}^{S*} = \begin{cases} \frac{4}{3}, & 0 \le \mu < 0.658466 \\ \frac{2\beta_1+1}{5\beta_1-1} = \frac{4-\mu}{3+\mu}, & 0.658466 \le \mu \le 1, \end{cases}$$

where at $\mu = 0.658466$, the demand $x_1 = \frac{4}{3}$ and $x_1 = \frac{4-\mu}{3+\mu}$ result in equal profits.

The optimal demands of informed trader with different horizons are illustrated in figure 1, the solid line represents the demand of trader with long-term horizon, the dashed line represents the demand of trader with short-term horizon. Firstly, informed trader with short-term horizon trades more aggressively than that with long-term horizon. Compared with long-term trader, short-term trader incurs price uncertainty at liquidation. The liquidation price is a random variable which depends on the trading intensity for the short-term trader, while for long-term trader, the liquidation price is the true payoff Since more intensive trading induces more information revelation, and drives the price closer to the true payoff The short-term trader thus increases the demand, in order to reduce the price uncertainty in the liquidation.

Secondly, both demands reduces with the probability of informed trading μ. The parameter μ captures the sensitivity of market maker to the total order, when μ is small, the market maker is less sensitive to the order flow, and the price adjustment is small. But when μ increases, the market maker becomes more sensitive to the order flow and the price impact

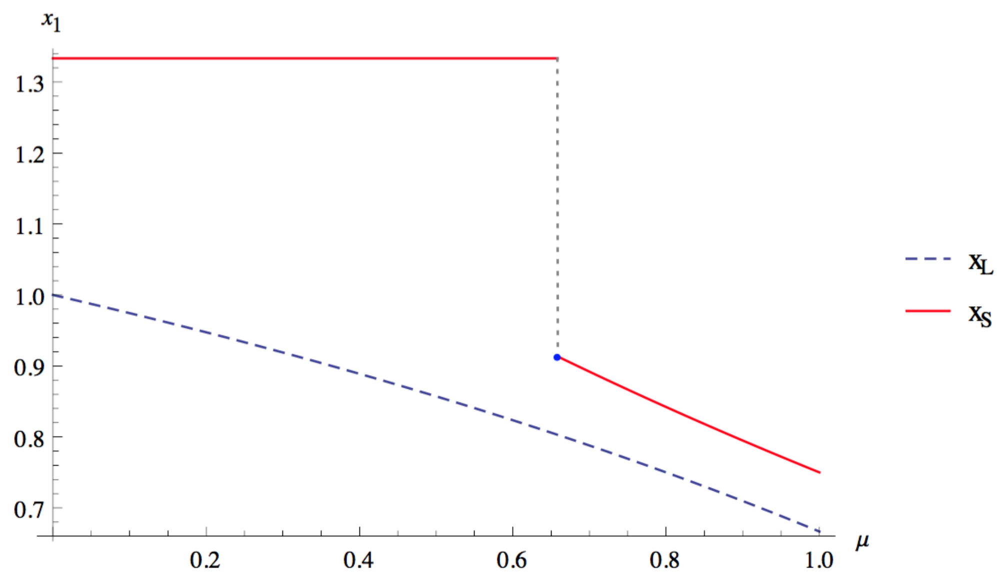


Figure 1: The demand of the informed trader

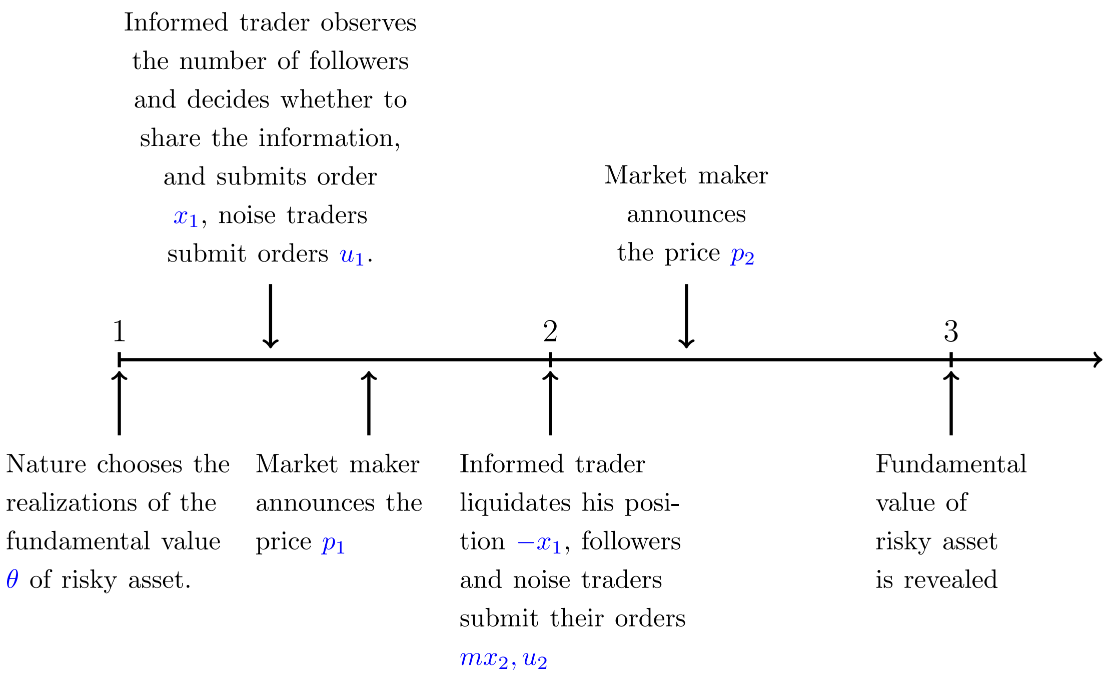
also increases. Therefore, the informed trader trades more cautiously. For long-term trader, the demand is always smaller than 1, which is the maximum demand from noise traders. But for short-term trader, the demand is larger than 1 but smaller than 2 when $\mu < 0.658466$. When μ is larger than 0.658466, the trader reduces the demand to below 1.

3 Information Disclosure

In this section, I derive the sub-game perfect equilibrium of the two-period trading game with information disclosure. The informed trader has the option to disclose the information to m other followers before the liquidation in the second period, $m \in \mathbb{Z}$ and $m \geq 1$. Each follower is rational, risk neutral, uninformed and does not trade in the first period. None of the follower has financial constraint. After receiving the private information, each follower trades strategically in the second period. Because the fundamental value will be revealed in the third period, each follower only trades one period. When $m \geq 2$, every follower trades competitively with each other.

The followers can be regarded as business partners or fellow investors who have connections with the informed trader, so the number of followers is observable by the informed trader. Knowing the number of followers, the informed trader can derive each follower's trading strategy and the expected price in the second period. The informed trader then compare the expected price with and without information disclosure and decides whether to disclose the information. The time line is shown in Figure 2.

Figure 2: Time line of the model



The equilibrium is defined as followers.

Definition 1. *An equilibrium is defined as a series of trading strategies X_1, X_2, prices P_1, P_2 and information disclosure strategy $\mathcal{D}$, such that the following conditions hold:*

(1) For given information disclosure strategy $\mathcal{D}$ and given number of followers m, the informed trader obtains highest expected profit with trading strategy X_1, that is,

$$E[\pi_1(X_1, \mathcal{D})|\theta] \quad \geq \quad E[\pi_1(X_1', \mathcal{D})|\theta].$$

(2) Conditional on information disclosure ($\mathcal{D} = 1$), each follower obtains highest expected profit with trading strategy X_2:

$$E[\pi_2(X_2, P_1)|\theta, \mathcal{D} = 1, P_1] \geq E[\pi_2(X_2', P_1)|\theta, \mathcal{D} = 1, P_1].$$

(3) The prices p_1 and p_2 are set as

$$p_1 = E[\theta|y_1],$$
$$p_2 = E[\theta|y_1, y_2].$$

(4) The information disclosure strategy $\mathcal{D} \in \{0, 1\}$ satisfies

$$\mathcal{D} \in \arg\max_{\mathcal{D}} E[\pi_1(X_1, \mathcal{D})|\theta].$$

The model is solved by backward induction. Firstly, I characterize the price equation in the second period, then I derive the follower's trading strategy. Secondly, I compute the trading strategy of the informed trader with information disclosure in the first period. At last, I derive the information disclosure decision of the informed trader.

3.1 Price in second period: with information disclosure

After receiving the information, each follower buys x_{2l} when $\theta = \theta_H$ and sells x_{2s} when $\theta = \theta_L$ in the second period. The informed trader liquidates the position, which is $-x_1$ when

$\theta = \theta_H$ and x_1 when $\theta = \theta_L$. The noise traders have order u_2. The total order is thus

$$y_2 = \begin{cases} mx_{2l} - x_1 + u_2 & \text{if} \quad y_1 = x_1 + u_1 \\ u_2 & \text{if} \quad y_1 = u_1 \\ -mx_{2s} + x_1 + u_2 & \text{if} \quad y_1 = -x_1 + u_1 \end{cases}$$

The price in the second period depends on the price p_1 and the total order flow y_2. Similar as the previous section, I only discuss the case when $\theta = \theta_H$, so two possible values of p_1 are considered : (1) $p_1 = p_M$; (2) $p_1 = p_0$.

(1) When $p_1 = p_M$:

(a) If $-x_1 \leq mx_{2l} - x_1 < 0$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad mx_{2l} - x_1 - 1 \leq y_2 < -1 \\ p_M & \text{if} \quad -1 \leq y_2 \leq mx_{2l} - x_1 + 1 \\ p_0 & \text{if} \quad mx_{2l} - x_1 + 1 < y_2 \leq 1 \end{cases} \tag{5}$$

(b) If $0 \leq mx_{2l} - x_1 < 2$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad 1 < y_2 \leq mx_{2l} - x_1 + 1 \\ p_M & \text{if} \quad mx_{2l} - x_1 - 1 < y_2 \leq 1 \\ p_0 & \text{if} \quad -1 \leq y_2 \leq mx_{2l} - x_1 - 1 \end{cases} \tag{6}$$

(c) If $mx_{2l} - x_1 \geq 2$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad mx_{2l} - x_1 - 1 < y_2 \leq mx_{2l} - x_1 + 1 \\ p_0 & \text{if} \quad -1 \leq y_2 \leq 1 \end{cases} \tag{7}$$

(2) When $p_1 = p_0$

(a) If $-x_{1l} \leq mx_{2l} - x_1 < 0$ and $-x_1 \leq mx_{2s} - x_1 < 0$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad mx_{2l} - x_1 - 1 \leq y_2 < -1 \\ p_M & \text{if} \quad -1 \leq y_1 \leq -1 - (mx_{2s} - x_1) \\ p_0 & \text{if} \quad -1 - (mx_{2s} - x_1) < y_1 < mx_{2l} - x_1 + 1 \\ \tilde{p}_M & \text{if} \quad mx_{2l} - x_1 + 1 \leq y_1 \leq 1 \\ \theta_L & \text{if} \quad 1 < y_1 \leq 1 - (mx_{2s} - x_1) \end{cases} \tag{8}$$

(b) If $0 \leq mx_{2l} - x_1 < 1$ and $0 \leq mx_{2s} - x_1 < 1$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad 1 < y_1 \leq 1 + mx_{2l} - x_1 \\ p_M & \text{if} \quad 1 - (mx_{2s} - x_1) \leq y_1 \leq 1 \\ p_0 & \text{if} \quad mx_{2l} - x_1 - 1 < y_1 < 1 - (mx_{2s} - x_1) \\ \tilde{p}_M & \text{if} \quad -1 \leq y_1 \leq mx_{2l} - x_1 - 1 \\ \theta_L & \text{if} - (mx_{2s} - x_1) - 1 \leq y_1 < -1 \end{cases} \tag{9}$$

(c) If $1 \leq mx_{2l} - x_1 < 2$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad 1 < y_1 \leq 1 + mx_{2l} - x_1 \\ p_M & \text{if} \quad mx_{2l} - x_1 - 1 \leq y_1 \leq 1 \\ p_0 & \text{if} \quad 1 - (mx_{2s} - x_1) < y_1 < mx_{2l} - x_1 - 1 \\ \tilde{p}_M & \text{if} \quad -1 \leq y_1 \leq 1 - (mx_{2s} - x_1) \\ \theta_L & \text{if} \quad -(mx_{2s} - x_1) - 1 \leq y_1 < -1 \end{cases} \tag{10}$$

(d) If $mx_2 - x_1 \geq 2$:

$$p_2 = \begin{cases} \theta_H & \text{if} \quad mx_2 - x_1 - 1 \leq y_1 \leq mx_2 - x_1 + 1 \\ p_0 & \text{if} \quad -1 \leq y_1 \leq 1 \\ \theta_L & \text{if} \quad -(mx_2 - x_1) - 1 \leq y_1 \leq 1 - (mx_2 - x_1) \end{cases}$$

There are two differences in the price functions p_2 with and without information disclosure. The first one lies in the informed trading intensity: without information disclosure, the total order y_2 contains the same information as the total order y_1, which is purely the order x_1. But with information disclosure, the informed part is $mx_2 - x_1$ in the second period, which is different from x_1. The information content depends on the total orders mx_2 from the followers, if the followers trade moderately, the total order $mx_2 - x_1$ might be smaller than x_1 and information disclosure results in less informed trading. If the followers trade aggressively such that $mx_2 - x_1$ is larger than x_1, the information disclosure results in more informed trading.

The second difference lies in the market maker's belief. Note that without information disclosure, the informed trader reverse her order in the second period, thus a negative order y_2 actually implies high fundamental value and a positive order y_2 implies low fundamental value. But with information disclosure, the followers trade on the information, and submit buy orders when the fundamental value is high and sell orders when the fundamental value is low, so a positive order y_2 implies high fundamental value and a negative order y_2 implies low fundamental value.

3.2 Second period: followers' strategies

In this section, I derive the follower's trading strategy in the second period. After receiving the information, the followers become informed and participate in the market from the second period. Since the fundamental value will be revealed in the third period, each follower only trades one period and tries to maximize the expected profit

$$E[\pi_2|\theta, p_1, x_1] = E[x_2(\theta - p_2)|\theta, p_1, x_1],$$

by taking the price function p_2 as given.

Similarly, I only consider the case when $\theta = \theta_H$. As derived in section 2.1 that when the fundamental value is high and when the informed trader appears in the market, the price p_1 takes three possible values: θ_H, p_M and p_0.

If $p_1 = \theta_H$, the information is fully revealed in the first period and there is no more trading in the second period.

If $p_1 = p_M$, the information is partially revealed in the first period. In the second period, each follower chooses x_{2l} to maximize the expected profit, by taking the other followers' demand into account and by taking the p_2 formulas (5)(6)(7) as given, which yields the expected profit function

$$E[\pi_2|\theta = \theta_H] = \begin{cases} x_{2l}\left[(\theta_H - p_M)\frac{x_{2l}+(m-1)x_2^*-x_1+2}{2}\right], & \text{if } 0 < x_{2l} \leq x_1 - (m-1)x_2^*, \\ x_{2l}\left[(\theta_H - p_M)\frac{2-x_{2l}-(m-1)x_2^*+x_1}{2}\right], & \text{if } x_1 - (m-1)x_2^* \leq x_{2l} \leq x_1 + 2 - (m-1)x_2^*, \\ 0, & \text{if } x_{2l} \geq x_1 - (m-1)x_2^*. \end{cases}$$

The expected profit of each follower depends on the informed trading intensity in the second period, that is, the value of $mx_{2l}-x_1$. Due to the competition, each follower trades by taking the demand from other followers, denoted by x_2^*, as given. In order to derive the optimal strategy of each follower, I first solve the constrained maximization problem in each interval, and get the demand function x_{2l}, which is a function of x_2^*. I then take $x_{2l} = x_2^*$ and derive the value of x_2^* in each interval. Next I substitute x_{2l} with the value of x_2^* and compute the implied profit in each interval. Then I compare the profits in each interval and derive the optimal demand x_2^*, which is the one results in the highest profit. The trading strategy of the follower is as follows.

Proposition 1. *If the price equals to p_M in the first period, conditional on information disclosure, each follower demands $x_2^* = \frac{x_1+2}{m+1}$.*

The proof is given in Appendix. On one hand, the optimal demand of each follower decreases with the number of followers m, that is due to the price impact. Each follower reduces the demand so that the total demand from all the followers is not too large, in order to hide the private information and eliminate the price impact. On the other hand, the total order from the followers $mx_2^* = \frac{m}{m+1}(x_1 + 2)$ increases with the number of m, that is due to the competition from the other followers, same result as Fishman and Hagerty (1995) and Foster and Viswanathan (1996).

At last, if $p_1 = p_0$, no information is revealed in the first period. Similar as the previous case, the expected profit function of each follower is given by

$$E[\pi_2|\theta=\theta_H] = \begin{cases} x_{2l}\left[(\theta_H-p_M)\frac{x_1-mx_{2s}}{2}+(\theta_H-p_0)\frac{x_{2l}+(m-1)x^*_{2l}-2x_1+mx_{2s}+2}{2}\right], & \text{if } 0<x_{2l}\le x_1-(m-1)x_2^*, \\ x_{2l}\left[(\theta_H-p_M)\frac{mx_{2s}-x_1}{2}+(\theta_H-p_0)\frac{2-mx_{2s}+2x_1-x_{2l}-(m-1)x^*_2}{2}\right], & \text{if } x_1-(m-1)x_2^*<x_{2l}\le \\ & x_1+1-(m-1)x_2^*, \\ x_{2l}\left[(\theta_H-p_M)\frac{2+x_1-x_{2l}-(m-1)x^*_{2l}}{2}\right], & \text{if } x_1+1-(m-1)x^*_{2l}<x_{2l}\le \\ & x_1+2-(m-1)x_2^*, \\ 0, & \text{if } x_{2l}>x_1-(m-1)x_2^*. \end{cases}$$

Each follower maximizes the expected profit by choosing the optimal demand x_{2l}, taking the price p_2 as given, and also taking the other $m-1$ followers' demands as given:

Proposition 2. *When the price equals to p_0 in the first period, conditional on information disclosure, an equilibria falls into the following categories:*

1. *Aggressive trading: each follower trades $x_2^*=\frac{x_1+2}{m+1}$ if $m\ge\frac{x_1+1}{2(1-\beta_1)}$.*
2. *Moderate trading: each follower trades $x_2^*=\frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ if $m<\frac{x_1+1}{2(1-\beta_1)}$.*

The proof is given in Appendix. The following graph illustrates the optimal demand of each follower when $p_1=p_0$.

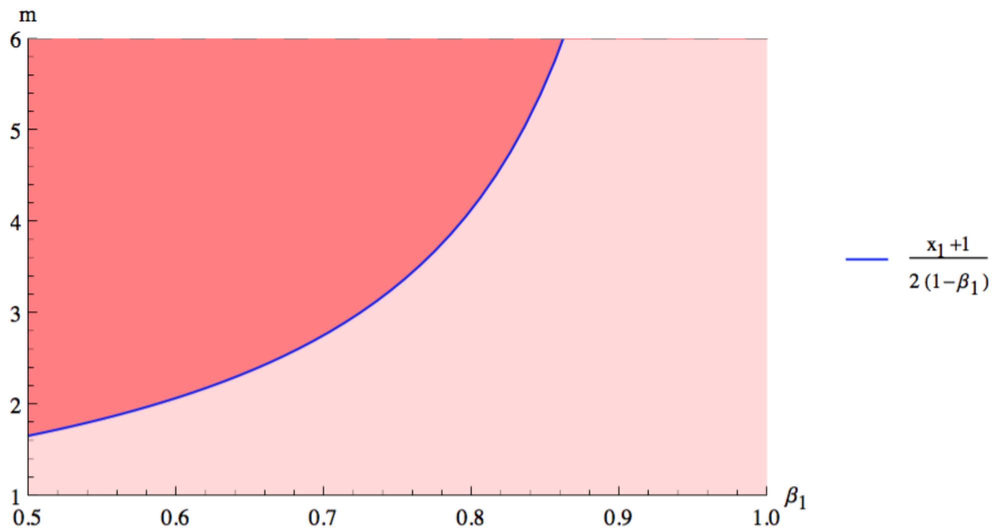


Figure 3: The follower's demand in the second period when $p_1 = p_0$

Figure 3 shows that the optimal demand x_2^* depends on the number of followers m and the probability of informed trading μ (note that $\beta_1 = \frac{1}{2-\mu}$). When $m \geq \frac{x_1+1}{2(1-\beta_1)}$, each follower trades aggressively and demand $x_2^* = \frac{x_1+2}{m+1}$. In this case, the total order in the second period is also aggressive and $1 \leq mx_2^* - x_1 < 2$. When $m < \frac{x_1+1}{2(1-\beta_1)}$, the competition among the followers becomes smaller, the probability of informed trading increases and market maker becomes more sensitive to the order, so each follower reduces the demand to $\frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$. The total order also reduces and $0 \leq mx_2^* - x_1 < 1$. In conclusion, the trading intensity of each follower is positive correlated with the number of followers m and negative correlated with the probability of informed trading μ.

Compared with the trading strategy when $p_1 = p_M$, the follower trades less intensively when $p_1 = p_0$. Note that the expected profit contains two parts: the order size x_2 and the marginal profit $\theta_H - p_2$. When $p_1 = p_M$, information has been partially revealed in the first period, the probability of more information revelation by trading in the second period is thud higher. So the price impact when $p_1 = p_M$ is higher and the marginal profit $\theta_H - p_2$ is thus lower for each follower. Therefore, the follower takes a larger demand by compensating the loss from the marginal profit. When $p_1 = p_0$, no information is revealed in the first period. The price impact is smaller in the second period and the marginal profit for each follower is higher, so the followers would rather to have a smaller position. But when the number of followers increases, the follower increases the demand due to more fierce competition.

3.3 Information disclosure strategy

When determining whether to disclose the information or not, the informed trader has to compare the expected price p_2 with and without information disclosure. Obviously, the optimal disclosure decision results in higher price p_2. Given the optimal demand of each follower and the pricing rule in the second period from in section 3.1 and 3.2, I then derive the information disclosure strategy of the informed trader. Without loss of generality, I impose one tie-breaking rule: whenever the informed trader is indifferent between disclosing and not disclosing the information, the informed trader strictly prefers disclosing the information.

In the first case, when $p_1 = p_M$, it is derived in section 2.2 that conditional on the appearance of informed trader and $\theta = \theta_H$, the price p_2 without information disclosure becomes

$$p_2 = \begin{cases} \theta_H & \text{if} \quad -1 \leq u_2 \leq x_1 - 1 \\ p_M & \text{if} \quad x_1 - 1 \leq u_2 \leq 1 \end{cases}$$

So the expected price without information disclosure is

$$E[p_2|\theta_H, x_1, p_1 = p_M, d = 0] = \frac{x_1}{2}\theta_H + (1 - \frac{x_1}{2})p_M, \tag{11}$$

where $d = 0$ denotes no information disclosure and $d = 1$ denotes information disclosure.

If the informed trader discloses the information, each follower trades $x_2^* = \frac{x_1+2}{m+1}$, the price p_2

becomes

$$p_2 = \begin{cases} \theta_H & \text{if} \quad 1 - (\frac{m(x_1+2)}{m+1} - x_1) < u_2 \leq 1 \\ p_M & \text{if} \quad -1 \leq u_2 \leq 1 - (\frac{m(x_1+2)}{m+1} - x_1) \end{cases}$$

and the expected price with information disclosure is

$$E[p_2|\theta_H, x_1, p_1 = p_M, d = 1] = \frac{2m - x_1}{2(m+1)}\theta_H + (1 - \frac{2m - x_1}{2(m+1)})p_M \tag{12}$$

Comparing equation (11) and (12), we find that the informed trader is better off disclosing the information when $x_1 \geq \frac{2m}{m+2}$, and she is better off not disclosing the information when $x_1 < \frac{2m}{m+2}$.

In the second case, when $p_1 = p_0$, the price p_2 conditional on no information disclosure is given by

$$p_2 = \begin{cases} \theta_H & \text{if} \quad -1 \leq u_2 < x_1 - 1 \\ p_M & \text{if} \quad x_1 - 1 \leq u_2 \leq 2x_1 - 1 \\ p_0 & \text{if} \quad 2x_1 - 1 < u_2 \leq 1 \end{cases}$$

which yields the expected price p_2 as

$$E[p_2|\theta_H, x_1, p_1 = p_0, d = 0] = \frac{x_1}{2}\theta_H + \frac{x_1}{2}p_M + (1 - x_1)p_0. \tag{13}$$

If the informed trader discloses the information, proposition 2 shows that each follower trades $x_1^* = \frac{x_1+2}{m+1}$ when $m \geq \frac{x_1+1}{2(1-\beta_1)}$ and trades $x_2^* = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ when $m < \frac{x_1+1}{2(1-\beta_1)}$. The price p_2 is given by

$$p_2 = \begin{cases} \theta_H & \text{if} \quad 1 - (\frac{m(x_1+\frac{1}{\beta_1})}{m+\frac{1}{2\beta_1}} - x_1) < u_2 \leq 1 \\ p_M & \text{if} \quad 1 - 2(\frac{m(x_1+\frac{1}{\beta_1})}{m+\frac{1}{2\beta_1}} - x_1) \leq u_2 \leq 1 - (\frac{m(x_1+\frac{1}{\beta_1})}{m+\frac{1}{2\beta_1}} - x_1) \\ p_0 & \text{if} \quad -1 \leq u_2 < 1 - 2(\frac{m(x_1+\frac{1}{\beta_1})}{m+\frac{1}{2\beta_1}} - x_1) \end{cases} \quad \text{or} \quad p_2 = \begin{cases} \theta_H & \text{if} \quad 1 - (\frac{m(x_1+2)}{m+1} - x_1) < u_2 \leq 1 \\ p_M & \text{if} \quad -1 \leq u_2 \leq 1 - (\frac{m(x_1+2)}{m+1} - x_1) \end{cases}$$

yields the expected price p_2 as

$$E[p_2|\theta_H, p_1 = p_0, m < \frac{x_1+1}{2(1-\beta_1)}, d=1] = \frac{2m-x_1}{4\beta_1 m+2}\theta_H + \frac{2m-x_1}{4\beta_1 m+2}p_M + (1-\frac{2m-x_1}{2\beta_1 m+1})p_0,$$
$$E[p_2|\theta_H, p_1 = p_0, m \geq \frac{x_1+1}{2(1-\beta_1)}, d=1] = \frac{2m-x_1}{2(m+1)}\theta_H + (1-\frac{2m-x_1}{2(m+1)})p_M. \quad (14)$$

Comparing the expected prices in equation (13) and (14), we see that the informed trader is better off disclosing the information when $x_1 \leq \frac{m}{m\beta_1+1}$, and she is better off not disclosing when $x_1 > \frac{m}{m\beta_1+1}$.

The information disclosure decision is contingent on the price p_1, which also depends on the value of x_1 and m. I summarize and illustrate the information disclosure strategy in the following figure.

Figure 4: The information disclosure decision of informed trader

Unconditional Disclosure means that the informed trader always discloses the information for any price p_1. *Conditional Disclosure* means the informed trader only discloses the information when $p_1 = p_M$ and not disclose the information when $p_1 = p_0$. *No Disclosure* means the informed trader does not disclose the information for any price p_1.

From Figure 4 we can see that if the informed trader initially decides to disclose the information in period one, she will keep the commitment and discloses the information in the second period whenever $p_1 = p_0$ or $p_1 = p_M$, if she trades $x_1 \in [0, \frac{m}{m\beta_1+1}]$; she will deviate to not disclose the information in the second period when $p_1 = p_0$ but keeps the commitment when $p_1 = p_M$, if she trades $x_1 \in (\frac{m}{m\beta_1+1}, \frac{2m}{m+2}]$; she will deviate for any price p_1 if she trades $x_1 > \frac{2m}{m+2}$ in the first period. Moreover, Figure 4 shows that the information disclosure is negative related to the demand of the informed trader: the information disclosure is more likely when the informed trader has a small demand in the first period.

3.4 The first period: the informed trader's strategy

In the first period, the informed trader tries to maximize the expected profit by choosing an optimal demand x_1. Section 3.3 shows that the information disclosure is a price contingent strategy, which also depends on the value of x_1. In the sub-game perfect equilibrium, the optimal demand x_1 is chosen by taking the information disclosure decision into account. Then I compute the expected profit function of the informed trader, which is a piecewise function. In general, there are four cases: (1) when $0 \le x_1 \le 2(1-\beta_1)m - 1$, the informed trader discloses the information whenever $p_1 \ne \theta_H$ and each follower trades $x_2 = \frac{x_1+2}{m+1}$ in the second period; (2) when $2(1-\beta_1)m - 1 < x_1 \le \frac{m}{m\beta_1+1}$, the informed trader discloses the information whenever $p_1 \ne \theta_H$, each follower trades $x_2 = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ if $p_1 = p_0$ and trades $x_2 = \frac{x_1+2}{m+1}$ if $p_1 = p_M$; (3) When $\frac{m}{m\beta_1+1} < x_1 \le \frac{2m}{m+2}$, the informed trader only discloses the information when $p_1 = p_M$ and each follower trades $x_2 = \frac{x_1+2}{m+1}$; (4) when $x_1 > \frac{2m}{m+2}$, the informed trader does not disclose the information in the second period.

I characterize the expected profit function of the informed trader by the value of m and μ. Since the expected profit is a piecewise function, when solving the optimal demand, I first solve the constrained maximization problem and get the optimal demand in each subdomain; then I substitute the optimal demand in the profit function and get the implied profit in each subdomain; in the end I compare the implied profit in each subdomain and get the maximum profit, as well as the corresponding demand x_1. The profit function and the optimal demand are given as follows.

- $\mathbf{m = 1}$

  At this point, $2(1-\beta_1)m - 1 < 0 < \frac{m}{m\beta_1+1} < \frac{2m}{m+1} < 1$, the expected profit of informed trader is

$$E[\pi_1|\theta_H] = \begin{cases} x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(2\beta_1 m+1)}\right] & \\ \quad +x_{1l}\left[(1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(p_M - p_0)\frac{2m-x_{1s}}{2(2\beta_1 m+1)}\right], & \text{if } 0 \le x_{1l} \le \frac{m}{m\beta_1+1}, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})[(\theta_H - p_M)\frac{x_{1l}}{2} + (p_M - p_0)\frac{x_{1s}}{2}]\right] & \text{if } \frac{m}{m\beta_1+1} < x_{1l} \le \frac{2m}{m+2}, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{x_{1l}}{2} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})[(\theta_H - p_M)\frac{x_{1l}}{2} + (p_M - p_0)\frac{x_{1s}}{2}]\right], & \text{if } \frac{2m}{m+2} < x_{1l} \le 1, \\ x_{1l}(1-\frac{x_{1l}}{2})(\theta_H - p_M)\frac{x_{1l}}{2}, & \text{if } 1 < x_{1l} \le 2. \end{cases}$$

The optimal demand is solved as

$$x_1^* = \begin{cases} \frac{4}{3}, & 0 \le \mu < 0.658466 \\ \frac{2\beta_1+1}{5\beta_1-1} = \frac{4-\mu}{3+\mu}, & 0.658466 \le \mu \le 1 \end{cases} \tag{15}$$

The optimal demand reduces with the probability of informed trading μ. The informed trader trades very aggressive when the probability of informed trading μ is small, and trades moderately when μ is large, at $\mu = 0.658466$, the expected profit of trading $\frac{4}{3}$ equals to the expected profit of trading $\frac{4-\mu}{3+\mu}$. Moreover, both $\frac{4}{3}$ and $\frac{4-\mu}{3+\mu}$ are larger than the disclosure threshold $\frac{2m}{m+2}$, so the informed trader does not disclose the information when $m = 1$. Indeed, the optimal demand is the same as the optimal demand derived in section 2.3, where there is no information disclosure.

- $\mathbf{m = 2}$

When $\mathbf{\frac{1}{2} \le \beta_1 \le \frac{3}{4}}$, $0 < 2(1-\beta_1)m - 1 < \frac{m}{m\beta_1+1} < \frac{2m}{m+1} = 1$, the expected profit is

$$E[\pi_1|\theta_H] = \begin{cases} x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)}\right] & \\ \quad +x_{1l}\left[(1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(p_M - p_0)\frac{2m-x_{1s}}{2(m+1)}\right] & \text{if } 0 \le x_{1l} \le 2m(1-\beta_1)-1, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(2\beta_1 m+1)}\right] & \\ \quad +x_{1l}\left[(1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(p_M - p_0)\frac{2m-x_{1s}}{2(2\beta_1 m+1)}\right] & \text{if } 2m(1-\beta_1)-1 < x_{1l} \le \frac{m}{m\beta_1+1}, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})[(\theta_H - p_M)\frac{x_{1l}}{2} + (p_M - p_0)\frac{x_{1s}}{2}]\right] & \text{if } \frac{m}{m\beta_1+1} < x_{1l} \le 1, \\ x_{1l}(1-\frac{x_{1l}}{2})(\theta_H - p_M)\frac{x_{1l}}{2}, & \text{if } 1 < x_1 \le 2. \end{cases}$$

When $\mathbf{\frac{3}{4}} < \beta_{\mathbf{1}} \leq \mathbf{1}$, $2(1-\beta_1)m - 1 \leq 0 < \frac{m}{m\beta_1+1} < \frac{2m}{m+1} = 1$, the expected profit is

$$E[\pi_1|\theta_H] = \begin{cases} x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(2\beta_1 m+1)}\right] & \\ \quad +x_{1l}\left[(1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(p_M-p_0)\frac{2m-x_{1s}}{2(2\beta_1 m+1)}\right] & \text{if } 0 \leq x_{1l} \leq \frac{m}{m\beta_1+1}, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})[(\theta_H-p_M)\frac{x_{1l}}{2} + (p_M-p_0)\frac{x_{1s}}{2}]\right] & \text{if } \frac{m}{m\beta_1+1} < x_{1l} \leq 1, \\ x_{1l}(1-\frac{x_{1l}}{2})(\theta_H-p_M)\frac{x_{1l}}{2}, & \text{if } 1 < x_1 \leq 2. \end{cases}$$

The optimal demand is

$$x_1^* = \begin{cases} \frac{4}{3}, & 0 \leq \mu < 0.405502 \\ \frac{-1+30\beta_1+16\beta_1^2-\sqrt{1-156\beta_1+580\beta_1^2+192\beta_1^3+256\beta_1^4}}{2(2+3\beta_1+8\beta_1^2)} & 0.405502 \leq \mu \leq 1, \end{cases} \tag{16}$$

We can see that when μ is small, the informed trader still trades very aggressively, even though the number of followers increases to 2. When μ becomes larger, the informed trader reduces the demand and trades moderately. At $\mu = 0.405502$, the expected profit of trading aggressively equals to the expected profit of trading moderately. Note that $\frac{-1+30\beta_1+16\beta_1^2-\sqrt{1-156\beta_1+580\beta_1^2+192\beta_1^3+256\beta_1^4}}{2(2+3\beta_1+8\beta_1^2)}$ is smaller than the disclosure threshold $\frac{m}{m\beta_1+1}$, which implies that the informed trader would disclose the information in the second period when μ is large.

- $\mathbf{m \geq 3}$

When $\mathbf{\frac{1}{2}} \leq \beta_{\mathbf{1}} \leq \mathbf{1} - \mathbf{\frac{1}{2m}}$, $0 \leq 2(1-\beta_1)m - 1 < \frac{m}{m\beta_1+1} < 1 < \frac{2m}{m+1} < 2$, the expected profit function is

$$E[\pi_1|\theta_H] = \begin{cases} x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)}\right] & \\ \quad +x_{1l}\left[(1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(p_M-p_0)\frac{2m-x_{1s}}{2(m+1)}\right] & \text{if } 0 \leq x_{1l} \leq 2m(1-\beta_1)-1, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(2\beta_1 m+1)}\right] & \\ \quad +x_{1l}\left[(1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})(p_M-p_0)\frac{2m-x_{1s}}{2(2\beta_1 m+1)}\right] & \text{if } 2m(1-\beta_1)-1 < x_{1l} \leq \frac{m}{m\beta_1+1}, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1-\frac{x_{1l}}{2}-\frac{x_{1s}}{2})[(\theta_H-p_M)\frac{x_{1l}}{2} + (p_M-p_0)\frac{x_{1s}}{2}]\right] & \text{if } \frac{m}{m\beta_1+1} < x_{1l} \leq 1, \\ x_{1l}(1-\frac{x_{1l}}{2})(\theta_H-p_M)\frac{2m-x_{1l}}{2(m+1)}, & \text{if } 1 < x_{1l} \leq \frac{2m}{m+2}, \\ x_{1l}(1-\frac{x_{1l}}{2})(\theta_H-p_M)\frac{x_{1l}}{2}, & \text{if } \frac{2m}{m+2} < x_{1l} \leq 2. \end{cases}$$

When $\mathbf{1 - \frac{1}{2m} < \beta_1 \leq 1}$, $2(1-\beta_1)m - 1 < 0 < \frac{m}{m\beta_1+1} < 1 < \frac{2m}{m+1} < 2$, the expected profit is

$$E[\pi_1|\theta_H] = \begin{cases} x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1 - \frac{x_{1l}}{2} - \frac{x_{1s}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(2\beta_1 m+1)}\right] & \\ \quad +x_{1l}\left[(1 - \frac{x_{1l}}{2} - \frac{x_{1s}}{2})(p_M - p_0)\frac{2m-x_{1s}}{2(2\beta_1 m+1)}\right] & \text{if } 0 < x_{1l} \leq \frac{m}{m\beta_1+1}, \\ x_{1l}\left[\frac{x_{1s}}{2}(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)} + (1 - \frac{x_{1l}}{2} - \frac{x_{1s}}{2})[(\theta_H - p_M)\frac{x_{1l}}{2} + (p_M - p_0)\frac{x_{1s}}{2}]\right] & \text{if } \frac{m}{m\beta_1+1} < x_{1l} \leq 1, \\ x_{1l}(1 - \frac{x_{1l}}{2})(\theta_H - p_M)\frac{2m-x_{1l}}{2(m+1)}, & \text{if } 1 < x_{1l} \leq \frac{2m}{m+2}, \\ x_{1l}(1 - \frac{x_{1l}}{2})(\theta_H - p_M)\frac{x_{1l}}{2}, & \text{if } \frac{2m}{m+2} < x_{1l} \leq 2. \end{cases}$$

The optimal demand is

$$x_1^* = \begin{cases} \frac{(m+1)(1+2\beta_1) - \sqrt{(1+2\beta_1)^2(m+1)^2 - 24(1-\beta_1)(m+1) + 48(1-\beta_1)^2}}{6(1-\beta_1)}, & 0 \leq \mu \leq \mu_m \\ \frac{(1-10\beta_1)m - (1+2\beta_1)(1+2\beta_1 m^2) + \sqrt{((1-10\beta_1)m - (1+2\beta_1)(1+2\beta_1 m^2))^2 - 32\beta_1 m(m+1)(1-5\beta_1-(1-\beta_1+4\beta_1^2)m)}}{2\left(1-5\beta_1-(1-\beta_1+4\beta_1^2)m\right)}, & \mu_m < \mu \leq 1, \end{cases} \tag{17}$$

where μ_m is the solution of

$2m(1-\beta_1) - 1 = \frac{(m+1)(1+2\beta_1) - \sqrt{(1+2\beta_1)^2(m+1)^2 - 24(1-\beta_1)(m+1) + 48(1-\beta_1)^2}}{6(1-\beta_1)}$. The optimal demand of the informed trader is still a decreasing function of μ. However, the optimal demand is not aggressive any more when μ is small. Instead, the informed trader trades moderately. Note that x_1 is smaller than the disclosure threshold $\frac{m}{m\beta_1+1}$ for any value of μ, which means that the informed trader discloses the information unconditionally when the number of follower is at least 3. Furthermore, at $\mu = \mu_m$, the optimal demand equals to $2(1-\beta_1)m - 1$ and is discontinuous at this point. That is because, the demand of each follower in the second period is discontinuous at this point. When $\mu \leq \mu_m$, each follower has an aggressive demand $\frac{x_1+2}{m+1}$, but when $\mu > \mu_m$, each follower reduces to a moderate demand $\frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ if $p_1 = p_0$. Considering the discontinuity of follower's demand, the informed trader also alter the demand at $\mu = \mu_m$.

Taking the demand x_1 as given by equation (15), (16) and (17), I plot the information disclosure strategy in Figure 5 with varying m and μ.

Figure 5: The information disclosure strategy

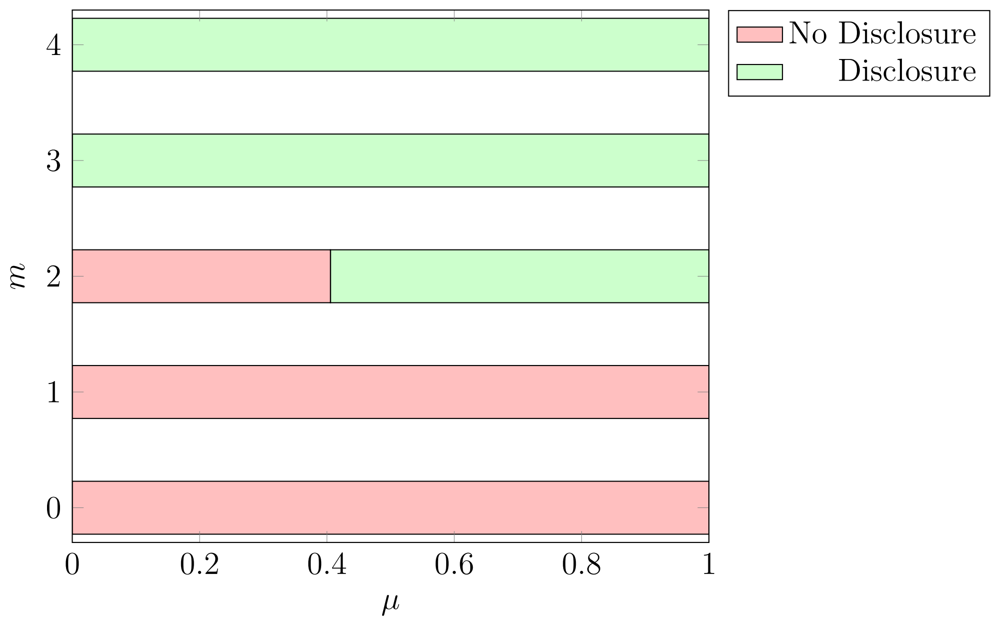
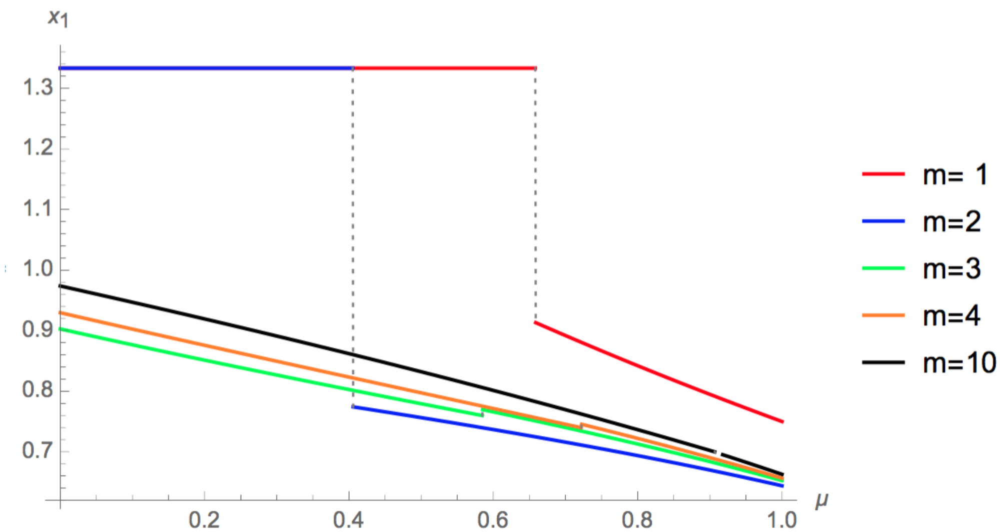


Figure 5 shows that when there is zero or one follower, the informed trader does not disclose the information. That is because the follow up capitals from the followers are not large enough if information is disclosed, the informed trader would rather signal the information to the market maker by the liquidation. When the number of followers increases to two, the follow up capitals increase due to the competition among the followers, the informed trader discloses the information, but conditional on the probability of informed trading μ. The informed trader discloses the information when μ is larger than 0.405502 and does not disclose the information when $\mu < 0.405502$. Note that the information disclosure is more likely when the demand from informed trader in the first period is small. So when μ is large, the information is more likely to be revealed through trading in the first period, the informed trader thus trades very cautiously. Therefore, she is more likely to disclose the information in the second period. But when μ is small, the informed trader can hide the information more easily and thus trades more intensively in the first period, so the information disclosure is less likely in the second period. When m is at least 3, the informed trader plays fully disclosure strategy. As documented by Foster and Viswanathan (1996) that when there are multiple informed traders, each trader trades more intensively and induces larger order flow. So when there are at least three followers, the informed trader is better off disclosing the information because the followers will bring more capital in the second period.

One more interesting question is how the informed trader adjust the demand with information disclosure and with different number of followers, I plot the optimal demand x_1^* for different m in Figure 6. Firstly, the demand x_1^* is an decreasing function of μ. Secondly, in terms of m, the demand when $m = 1$ is the same with the benchmark, since there is no information disclosure when $m = 1$. When m increases to 2, the informed traders reduces the demand conditional on information disclosure, as shown by the blue line. When m is larger than 3, the informed trader always discloses the information. On one hand, the demand becomes less aggressively, that is, $x_1^* < 1$. On the other hand, the demand increases with the m. That is because when the number of followers m increases, high competition in the second period induces aggressive demand and thus more information revelation to the market maker, and the price is more close to the fundamental value. When the price uncertainty becomes smaller, the informed trader is more brave to trade intensively in the first period.

Figure 6: The informed trader's order



To sum up, the equilibrium with information disclosure is given as follows:

Proposition 3. *Conditional on $\theta = \theta_H$: (1) when there are no follower or only one follower, the informed trader does not disclose the information, the demand x_1 is given by equation (15); (2) when there are two followers, the informed trader discloses the information when $\mu \geq 0.405502$, each follower trades $x_2 = \frac{x_1+2}{m+1}$. The informed trader does not disclose the information when $\mu < 0.405502$, the demand x_1 of informed trader is given by equation (16); (3) when there are at least three followers, the informed trader discloses the information regardless of μ, the demand x_1 is given by equation (17). Each follower trades $x_2 = \frac{x_1+2}{m+1}$ when $p_1 = p_M$ or when $p_1 = p_0$ and $0 \leq x_1 \leq 2m(1-\beta_1) - 1$, and trades $x_2 = \frac{x_1 + \frac{1}{\beta_1}}{m + \frac{1}{2\beta_1}}$ when $p_1 = p_0$ and $x_1 > 2m(1-\beta_1) - 1$.*

3.5 Information disclosure: when the number of followers is uncertain

In this section, I extend the model by assuming that the number of follower m is uncertain. Suppose that the number of m is only observable in the second period. Then the informed trader will postpone the disclosure decision to the second period. However, she still takes the disclosure strategy into account when she makes the trading strategy in the first period. Assuming that the number m follows Poisson distribution, $m \sim \mathcal{P}(\lambda)$, so the expected number of followers is λ. Then the optimal demand in the first period, denoted by $\tilde{x}_1$, is a weighted average of the demand $x_1(m)$, which is given by

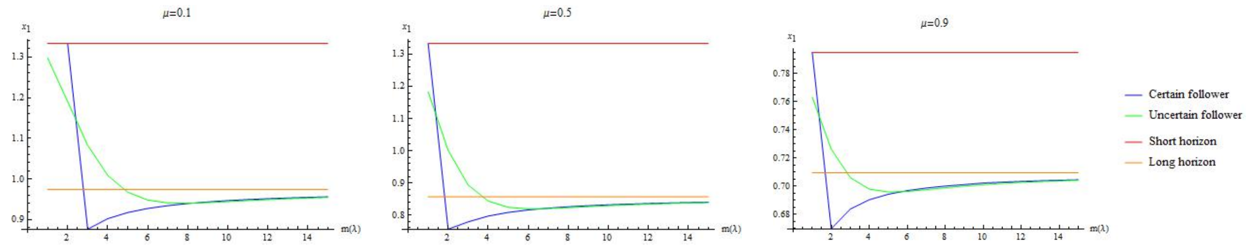
$$\tilde{x}_1 = \sum_{k=0}^{+\infty} x_1(k) * P(m = k) = \sum_{k=0}^{+\infty} \frac{x_1(k) * \lambda^k * e^{-\lambda}}{k!}.$$

The demand $\tilde{x}_1$ is depicted by the green line in figure 7.

4 Implication of information disclosure

In this section I discuss the implication of information disclosure, focusing on the impact of information disclosure on the trading intensities, price discovery and the profits of market participants.

First of all, I examine the effect of information disclosure to the intensity of informed orders. The effect is obvious in the second period: the information disclosure induces more informed trading, because the number of informed traders increases and high competition induces intensive trading. However, the effect in the first period is not very clear. For comparison, I plot the optimal demand of informed trader under four different situations in figure 7 : (1)when the informed trader has short horizon but not disclose the information, depicted by the red line; (2)when the informed investor has short horizon and has the option to disclose the information, the number of followers is observable in the first period, depicted by the blue line; (3)when the informed trader has short horizon, has the option to disclose the information, and the number of followers is unobservable in the first period, depicted by green line; (4)when the informed investor has long horizon, as depicted by the orange line.



(a) $\mu = 0.1$ (b) $\mu = 0.5$ (c) $\mu = 0.9$

Figure 7: The demand of informed trader under different situations

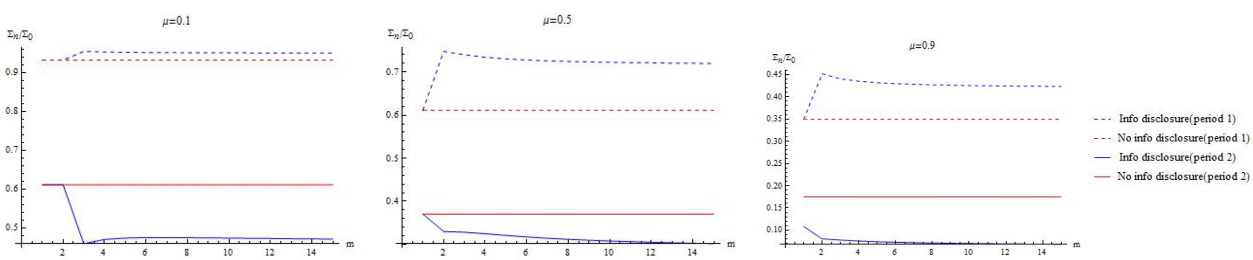
We observe that when the informed investor has short horizon, she tends to trade more aggressively, compared to the situation with long horizon. The excessive demand comes from the price uncertainty in the second period, since the informed trader needs to signal the identity to the market maker when she liquidates the position. But with long horizon and no price uncertainty in the second period, the informed trader would trade moderately because she needs to prevent the information leakage in the first period.

With the option to disclose the information, the informed investor reduces her demand gradually with the (expected) number of followers. When the number of followers are large enough, the optimal demand converges to the optimal demand when the informed trader has long horizon. The demand reduction comes from the elimination of price uncertainty with large number of followers. Interestingly, compare the green line with the blue line, we can see that the informed trader trades more when she is uncertain about the number of followers. When the informed trader knows the number of followers, she will trade even less than the trader with long horizon.

Secondly, I study the effect on price discovery. As defined in Kyle (1985), price discovery measures the informativeness of the price, that is, how much information is revealed in the price. The price discovery is computed as the ex-post variance of the fundamental value:

$$\Sigma_1 = E[Var[\theta|y_1]], \qquad \Sigma_2 = E[Var[\theta|y_1, y_2]].$$

Small value of Σ_t indicates more informativeness of the price p_t. To show the price discovery implication on information disclosure, I plot the ratio of ex-post variance to the ex-ante variance of the fundamental value for each period in figure 8. Red lines represent the case with no information disclosure, blue lines represent the case with information disclosure. Dotted lines represent the ratio $\frac{\Sigma_1}{\Sigma_0}$, solid lines represent the ratio $\frac{\Sigma_2}{\Sigma_0}$.



(a) $\mu = 0.1$ (b) $\mu = 0.5$ (c) $\mu = 0.9$

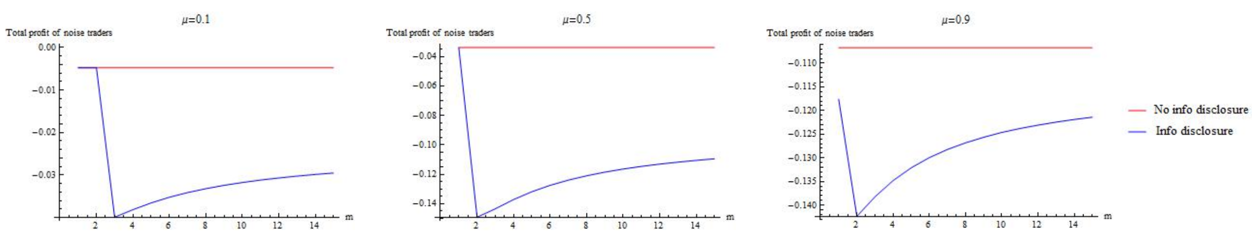
Figure 8: The measure of price discovery

First of all, figure 8 shows that the price efficiency increases with trading: $\Sigma_2 < \Sigma_1 < \Sigma_0$. More interestingly, the impact of information disclosure on the price discoveries are opposite in the two period. In period one, the information disclosure reduces the price discovery while in period two it increases the price discovery. This effect is the same with the findings in the forced information disclosure literatures such as Yang and Zhu (2017) and Steven Huddart (2001). But the reasons of the reduction of price discovery in the first period are different. When the informed trader is enforced to disclose the information, she plays a mixed strategy by adding noise into the demand. In this paper the informed trader plays a pure strategy but she reduces the demand when she decides to disclose the information voluntarily.

Next, I study the effect on the profits of the market participants. The effect is obvious for the market maker, the informed trader and the followers. The market maker is competitive and always get zero expected profit, so there is no impact on the market maker. The expected profit of informed trader is increased by the information disclosure since the disclosure is an endogenous decision of the informed trader. The information disclosure is also beneficial to the followers, because the disclosure results in the market participation and the positive expected profit for each follower. The impact on the noise traders is not clear. I compute the expected profit of the noise traders in each period:

$$E[\pi_1^n] = E[u_1(v - p_1)], \qquad E[\pi_2^n] = E[u_2(v - p_2)].$$

Then plot the total profit of the noise traders in figure 9, red line represents the case with no information disclosure, the blue line represents the case with information disclosure.



(a) $\mu = 0.1$ (b) $\mu = 0.5$ (c) $\mu = 0.9$

Figure 9: The expected total profit of noise traders

The expected profits of the noise traders are negative, which is consistent with the asymmetric information models that the informed traders profit from the loss of noise traders. Moreover, noise traders incur more loss with information disclosure. Since the overall informed trading increases with information disclosure, the market therefore becomes less liquid, and the noise traders lose more.

The implications of the information disclosure are summarized in the following proposition, the superscripts "d" denotes information disclosure and "nd" denotes no information disclosure.

Proposition 4. *In the two-period model, the information disclosure option reduces the trading intensity in the first period, increases price discovery in the second period but reduces price discovery in the first period, information disclosure also harms noise traders:*

$$
\begin{aligned}
x_1^d &\leq x_1^{nd}, \\
\Sigma_1^d &> \Sigma_1^{nd}, \quad \Sigma_2^d < \Sigma_2^{nd}, \\
E[\pi_n^d] &< E[\pi_n^{nd}].
\end{aligned}
$$

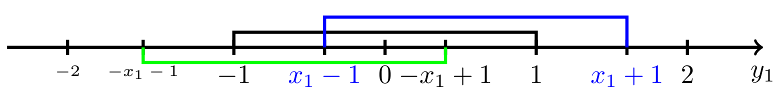
5 Conclusion

In this paper, I studied the incentive of information disclosure from large informed investors. The informed investor has short horizon and can not stay invested until the fundamental value of risky asset be publicly revealed. I show that the informed investor strategically disclose the information to the financial market. Endogenous information disclosure from the informed investor provides a way to eliminate the price uncertainty and accelerate the correction of mis-pricing. Information disclosure is more likely when the scope for private information is large, and when the informed investor has high reputation and her information can be easily spread and taken by other investors. More interestingly, I found that endogenous information disclosure harms price discovery before the disclosure, since the informed investor trades less aggressively. Information disclosure is also detrimental to noise traders since the expected loss of noise traders increases.

Appendices

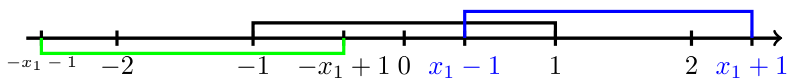
A Pricing functions in section 2.1

The total order flow in the first period is either $y_1 = x_1 + u_1$, when the fundamental value is high and when the informed trader appears in the market, which happens with probability $\beta_0\mu$. The total order flow may also be $y_1 = u_1$ when no informed trader appears, which happens with probability $1-\mu$. Or the total order be $y_1 = -x_1 + u_1$, when the fundamental value is low and the informed trader appears in the market, which happens with probability $(1-\beta_0)\mu$. The following figure plots the possible values of y_1 when $0 \le x_1 < 1$. Blue line depicts $y_1 = x_1 + u_1$, green line depicts $y_2 = -x_1 + u_1$ and the black line depicts $y_1 = u_1$.



We can see from the figure that when $1 < y_1 \le x_1 + 1$, the total order y_1 could only be x_1+u_1, which implies that the fundamental value is high, so the market maker sets the price $p_1 = \theta_H$. When $1 - x_1 < y_1 \le 1$, the order y_1 could either be $x_1 + u_1$ or be u_1. Conditional on $y_1 = x_1 + u_1$, the probability of high fundamental value is 1, conditional on $y_1 = u_1$, the probability of high fundamental value is β_0. Thus, the probability of high fundamental value conditional on observing $1 - x_1 < y_1 \le 1$ is $\beta_1 = \frac{\beta_0\mu+\beta_0(1-\mu)}{\beta_0\mu+(1-\mu)} = \frac{1}{2-\mu}$, then the market maker sets the price $p_1 = p_M \equiv \beta_1\theta_H + (1-\beta_1)\theta_L$. When $x_1 - 1 \le y_1 \le 1 - x_1$, the total order flow could be either x_1+u_1, or u_1 or $-x_1+u_1$, the market maker's belief will not be updated since no new information is revealed, and the price $p_1 = p_0$. When $-1 \le y_1 < x_1 - 1$, the total order flow could be either $-x_1 + u_1$ or be u_1. Conditional on $y_1 = -x_1 + u_1$, the probability of high fundamental value is 0, conditional on $y_1 = u_1$, the probability of high fundamental value is β_0. So after observing $-1 \le y_1 < x_1 - 1$, the market maker updates the belief that the probability of high fundamental value as $\frac{\beta_0(1-\mu)}{(1-\beta_0)\mu+1-\mu} = \frac{1-\mu}{2-\mu} = 1-\beta_1$, and sets the price $p_1 = \tilde{p}_M \equiv (1-\beta_1)\theta_H + \beta_1\theta_L$. When $-x_1 - 1 \leq y_1 < -1$, the total order flow y_1 could only come from $-x_1 + u_1$, which implies that the fundamental value is low and the price $p_1 = \theta_L$.

When the value of x_1 increases and $1 \leq x_1 < 2$, $x_1 + u_1$ moves towards the right and $-x_1 + u_1$ moves towards the left and there is no intersection between $x_1 + u_1$ and $-x_1 + u_1$. Similar as the previous case, we determine the price p_1 in five different intervals.



When the value x_1 further increases and $x_1 \geq 2$, there is no intersection among the three cases. The information is fully revealed in the total order, so $p_1 = \theta_H$ when $x_1 - 1 \leq y_1 \leq x_1 + 1$, and $p_1 = \theta_L$ when $-x_1 - 1 \leq y_1 \leq -x_1 + 1$ and $p_1 = p_0$ when $-1 \leq y_1 \leq 1$.

B Pricing functions in section 2.2

The price in the second period is the expected fundamental value conditional on the historical and current order flows and $p_2 = E[\theta|y_1, y_2] = E[\theta|p_1, y_2]$. The price p_2 depends on the value of p_1 and the y_2, we discuss the following five situations:

1. $p_1 = p_0$ and $0 \leq x_1 < 1$.

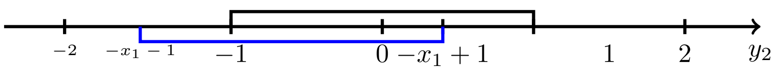
   In this case, no information is revealed in the first period, the market maker's belief is not updated. Then in the second period, the pricing rule is the same with the one in the first period, except that now a negative order indicates high fundamental value and a positive order indicates low fundamental value.

2. $p_1 = p_0$ and $1 \leq x_1 < 2$.

   From the pricing function 2 we know that when $1 \leq x_1 < 2$, $p_1 = p_0$ only happens when the market maker recognizes that total order is from the noise traders. Then in the second period the total order $y_2 = u_2$ and still no more information been revealed, the price $p_2 = p_1 = p_0$.

3. $p_1 = p_M$ and $0 \leq x_1 < 1$.

   If $p_1 = p_M$, the market maker recognized that the total order flow in the first period is either $y_1 = x_1 + u_1$ or $y_1 = u_1$. Then in the second period, the total order will be either $y_2 = -x_1 + u_2$ or $y_2 = u_2$. I plot the total order flow y_2 in the following figure. The blue line depicts $y_2 = -x_1 + u_2$ and the black line depicts $y_2 = u_2$.



   The above figure shows that if $-x_1 - 1 \leq y_2 < -1$, the total order $y_2 = -x_1 + u_2$ and the information is fully revealed, so the market maker sets $p_2 = \theta_H$. If $-1 \leq y_2 < 1 - x_1$, the total order could either be $-x_1 + u_2$ or u_2, the market maker can not recognize the where the order come from and no more information is revealed, so $p_2 = p_1 = p_M$. If $1 - x_1 \leq y_2 \leq 1$, the market maker recognizes that the total order comes from noise traders, then she updates her belief and sets $p_2 = p_0$.

4. $p_1 = p_M$ and $1 \leq x_1 < 2$.

   This case is the same with the previous one.

5. $p_1 = \tilde{p}_M$ and $0 \leq x_1 < 2$.

   This case is similar as the one when $p_1 = p_M$ and $0 \leq x_1 < 2$, since the market maker recognizes that the total order in the first period could be either $y_1 = -x_1 + u_1$ or $y_1 = u_1$. Then the total order in the second period is either $y_2 = x_1 + u_2$ or $y_2 = u_2$. The price will be

$$p_2 = \begin{cases} \theta_L & \text{if} \quad 1 < y_2 \leq x_1 + 1 \\ \tilde{p}_M & \text{if} \quad x_1 - 1 \leq y_2 \leq 1 \\ p_L & \text{if} \quad -1 < y_2 < x_1 - 1 \end{cases}$$

C Proof of Proposition 1

Proof. The expected profit of one follower, say a, by taking other $m-1$ followers' demand as given is as follows

$$E[\pi_2|\theta=\theta_H]=\begin{cases} x_2\left[(\theta_H-p_M)\frac{x_2+(m-1)x_2^*-x_1+2}{2}\right], & \text{if } 0<x_2\leq x_1-(m-1)x_2^*,\\ x_2\left[(\theta_H-p_M)\frac{2-x_2-(m-1)x_2^*+x_1}{2}\right], & \text{if } x_1-(m-1)x_2^*\leq x_2\leq x_1+2-(m-1)x_2^*,\\ 0, & \text{if } x_2\geq x_1-(m-1)x_2^*,\end{cases}$$

where x_2^* is the optimal demand of other follower. In the first case, when the total orders from all the followers is very small, that is when $0<(m-1)x_2^*+x_2<x_1$, the expected profit of follower a is an increasing function. In the second interval, when the total order from all the followers is larger than x_1 but smaller than x_1+2, the expected profit of follower a is concave and the optimal demand is attained at $x_2=\frac{x_1+2-(m-1)x_2^*}{2}$. Equalizing x_2 with x_2^* yields the optimal demand as $x_2^*=\frac{x_1+2}{m+1}$. Moreover, x_2^* has to satisfy the boundary condition, which is $x_1\leq mx_2^*\leq x_1+2$. On the left side, $x_1\leq\frac{m(x_1+2)}{m+1}$ requires $x_1\leq 2m$, which is satisfied since $x_1\leq 1$ and $m\geq 1$. On the right side, $\frac{m(x_1+2)}{m+1}\leq x_1+2$ is also satisfied for any $m\geq 1$. In conclusion, the optimal demand of each follower in the second period when $p_1=p_M$ is $x_2^*=\frac{x_1+2}{m+1}$.

□

D The proof of proposition 2

Conditional on information disclosure, there will be $m\geq 2$ followers competing with each other in the second period. I first derive the optimal reaction function of one follower, say follower a, by taking other $m-1$ followers' order as given. Since the followers are homogeneous, then I equalize the demand of follower a with other $m-1$ followers and derive the optimal demand.

The expected profit function of follower a is given as follows

$$E[\pi_2|\theta=\theta_H]=\begin{cases} x_{2l}\left[(\theta_H-p_M)\frac{x_1-mx_{2s}}{2}+(\theta_H-p_0)\frac{x_{2l}+(m-1)x_{2l}^*-2x_1+mx_{2s}+2}{2}\right], & \text{if } 0<x_{2l}\leq x_1-(m-1)x_2^*,\\ x_{2l}\left[(\theta_H-p_M)\frac{mx_{2s}-x_1}{2}+(\theta_H-p_0)\frac{2-mx_{2s}+2x_1-x_{2l}-(m-1)x_2^*}{2}\right], & \text{if } x_1-(m-1)x_2^*<x_{2l}\leq \\ & x_1+1-(m-1)x_2^*,\\ x_{2l}\left[(\theta_H-p_M)\frac{2+x_1-x_{2l}-(m-1)x_2^*}{2}\right], & \text{if } x_1+1-(m-1)x_2^*<x_{2l}\leq \\ & x_1+2-(m-1)x_2^*,\\ 0, & \text{if } x_{2l}>x_1-(m-1)x_2^*, \end{cases}$$

where x_2^* denotes the optimal demand of other $m-1$ followers. The expected profit is a piecewise function, which depends on the size of the total order. I compute the optimal reaction function of follower a in each subdomain respectively.

1) When $\mathbf{0\leq x_{2l}+(m-1)x_2^*\leq x_1}$, the expected profit of follower a is an increasing function in this subdomain. Therefore, the follower a takes $x_{2l}=x_1-(m-1)x_2^*$.

2) When $\mathbf{x_1<x_{2l}+(m-1)x_2^*\leq x_1+1}$, the expected profit of follower a is given by $x_{2l}\left[(\theta_H-p_M)\frac{mx_{2s}-x_1}{2}+(\theta_H-p_0)\frac{2-mx_{2s}+2x_1-x_{2l}-(m-1)x_{2l}^*}{2}\right]$. The optimal demand is derived from the first order condition, which yields that $x_{2l}=x_{2s}=\frac{2+2\beta_1x_1-2\beta_1(m-1)x_2^*}{2\beta_1+1}$. Moreover, the boundary condition $x_1<x_{2l}+(m-1)x_2^*\leq x_1+1$ should be satisfied: on the left side, $x_1<x_{2l}+(m-1)x_2^*$ requires that $x_1<2+(m-1)x_2^*$, the left side boundary is satisfied since $x_1<1$; on the right side, $x_{2l}+(m-1)x_2^*\leq x_1+1$ requires that $x_1\geq(m-1)x_2^*+1-2\beta_1$. So the expected profit is either a concave function when $x_1\geq(m-1)x_2^*+1-2\beta_1$ or an increasing function when $x_1<(m-1)x_2^*+1-2\beta_1$. Combining the result from case (1), we further derive that the profit function of follower a is concave in the subdomain $0\leq x_{2l}\leq x_1+1-(m-1)x_2^*$, when $x_1\geq(m-1)x_2^*+1-2\beta_1$,

and follower a takes demand $x_{2l} = x_{2s} = \frac{2+2\beta_1 x_1 - 2\beta_1(m-1)x_2^*}{2\beta_1+1}$. Or the expected profit function is an increasing function in the subdomain $0 \leq x_{2l} \leq x_1 + 1 - (m-1)x_2^*$, when $x_1 < (m-1)x_2^* + 1 - 2\beta_1$, and follower a takes demand $x_{2l} = x_{2s} = x_1 + 1 - (m-1)x_2^*$.

3) When $\mathbf{x_1 + 1 < x_{2l} + (m-1)x_2^* \leq x_1 + 2}$, the expected profit function of follower a is $x_{2l}\left[(\theta_H - p_M)\frac{2+x_1-x_{2l}-(m-1)x_2^*}{2}\right]$. Taking the first order condition yields the optimal demand as $x_{2l} = \frac{x_1+2-(m-1)x_2^*}{2}$. Then we check whether the boundary condition is satisfied: on the left side, $x_1 + 1 < x_{2l} + (m-1)x_2^*$ requires that $x_1 < (m-1)x_2^*$; on the right side, $x_{2l} + (m-1)x_2^* \leq x_1 + 2$ requires that $(m-1)x_2^2* < x_1 + 2$, which is satisfied since $(m-1)x_2^* < x_1 + 2 - x_{2l}$. So the expected profit function in this subdomain is either concave when $x_1 < (m-1)x_2^*$, or decreasing when $x_1 \geq (m-1)x_2^*$.

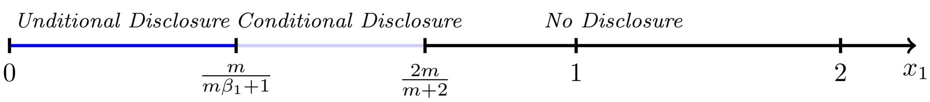
Combining the result from case (2) and (3), we conclude that, the expected profit function of follower a is either (a) concave function and takes maximum value at $x_{2l} = \frac{x_1+2-(m-1)x_2^*}{2}$, when $x_1 \leq (m-1)x_2^* + 1 - 2\beta_1$. Taking $x_{2l} = x_2^*$ gives the value of $x_2^* = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$, and the boundary condition as $x_1 \leq 2(1-\beta_1)m - 1$; or (b) concave and takes maximum value at $x_{2l} = \frac{2+2\beta_1 x_1 - 2\beta_1(m-1)x_2^*}{2\beta_1+1}$ when $x_1 > (m-1)x_2^*$. Taking $x_{2l} = x_2^*$ gives the value of $x_2^* = \frac{x_1+2}{m+1}$, and the boundary condition as $x_1 > \frac{m-1}{\beta_1+\frac{1}{2}}$; or (c) hump function when $2(1-\beta_1)m - 1 < x_1 \leq \frac{m-1}{\beta_1+\frac{1}{2}}$ and takes maximum value at either $x_{2l} = x_2^* = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ or $x_{2l} = x_2^* = \frac{x_1+2}{m+1}$. However, comparing the expected profit at the two maximum points, we can see that the profit at $x_{2l} = x_2^* = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$, which is $\frac{1-\beta_0}{2}(\frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}})^2$, is larger than the expected profit at $x_{2l} = x_2^* = \frac{x_1+2}{m+1}$, which is $\frac{1-\beta_1}{2}(\frac{x_1+2}{m+1})^2$, when $2(1-\beta_1)m - 1 < x_1 \leq \frac{m-1}{\beta_1+\frac{1}{2}}$. so the maximum profit is obtain at $x_{2l} = x_2^* = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ in the last case.

In conclusion, each follower trades $x_2^* = \frac{x_1+2}{m+1}$ when $0 < x_1 \leq 2(1-\beta_1)m - 1$ and trades $x_2^* = \frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ when $2(1-\beta_1)m - 1 < x_1 \leq 1$.

E The expected profit of informed trader in section 3.4

I calculate the expected profit of the informed trader by the order showed in Figure 4.There are 5 cases , depending on the value of m and β_1:

1) $\mathbf{m=1}$

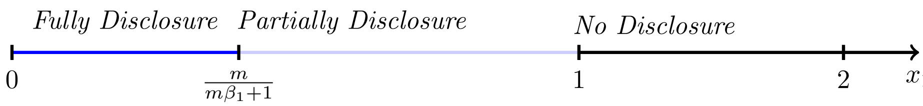


In this case, $2(1-\beta_1)m-1<0<\frac{m}{m\beta_1+1}<\frac{2m}{m+2}<1$. If $x_1\in[0,\frac{m}{m\beta_1+1}]$, the informed trader will discloses the information in the second period as long as $p_1\neq\theta_H$. Each follower will trade $x_2=\frac{x_1+2}{m+1}$ if $p_1=p_M$ and $x_2=\frac{x_1+\frac{1}{\beta_1}}{m+\frac{1}{2\beta_1}}$ if $p_1=p_0$ in the second period. So the expected profit of the informed trader is $\frac{1}{4}x_{1l}\left[\frac{(-x_{1l}+x_{1s}+2)((\beta_1-\beta_0)(2m+x_{1s})+(1-\beta_0)(2m-x_{1l}))}{2\beta_1 m+1}-\frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right](\theta_H-\theta_L)$. If $x_1\in(\frac{m}{m\beta_1+1},\frac{2m}{m+2}]$, the informed trader will disclose the information if $p_1=p_M$ and each follower will trade $x_2=\frac{x_1+2}{m+1}$ in the second period, and the informed trader will not disclose the information if $p_1=p_0$. So the expected profit is $\frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0))-\frac{x_{1s}(1-\beta_1)(2m-x_{1l})}{m+1}\right](\theta_H-\theta_L)$. If $x_1\in(\frac{2m}{m+2},2)$, the informed trader does not disclose the information and the expected profit is given as $\frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0))-(1-\beta_1)x_{1l}x_{1s}\right](\theta_H-\theta_L)$ when $x_1\leq 1$ and $\frac{1}{4}(1-\beta_1)x_{1l}^2(2-x_{1l})$ when $x_1>1$. In conclusion, the expected profit function is

$$\frac{E[\pi_1|\theta=\theta_H]}{\theta_H-\theta_L}=\begin{cases}\frac{1}{4}x_{1l}\left[\frac{(-x_{1l}+x_{1s}+2)((\beta_1-\beta_0)(2m+x_{1s})+(1-\beta_0)(2m-x_{1l}))}{2\beta_1 m+1}-\frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right] & \text{if } 0\leq x_1\leq\frac{m}{m\beta_1+1},\\ \frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0))-\frac{x_{1s}(1-\beta_1)(2m-x_{1l})}{m+1}\right] & \text{if } \frac{m}{m\beta_1+1}\leq x_1\leq\frac{2m}{m+2},\\ \frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0))-(1-\beta_1)x_{1l}x_{1s}\right], & \text{if } \frac{2m}{m+2}\leq x_1\leq 1,\\ \frac{1}{4}(1-\beta_1)x_{1l}^2(2-x_{1l}), & \text{if } 1\leq x_1\leq 2.\end{cases}$$

2) $\mathbf{m = 2}$ && $\mathbf{\frac{3}{4}} < \beta_\mathbf{1} \leq \mathbf{1}$

In this case, $2(1-\beta_1)m-1 < 0 < \frac{m}{m\beta_1+1} < \frac{2m}{m+1} = 1$. It's the same with case 1) except

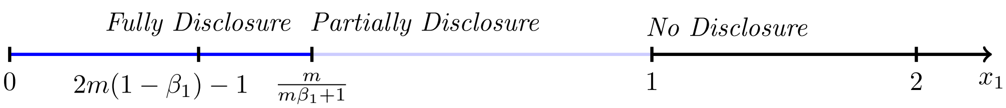


that the interval $[\frac{2m}{m+2}, 1)$ does not exist since $\frac{2m}{m+2} = 1$ when $m = 1$. The expected profit function is given as

$$\frac{E[\pi_1|\theta=\theta_H]}{\theta_H-\theta_L} = \begin{cases} \frac{1}{4}x_{1l}\left[\frac{(2-x_{1l}+x_{1s})((\beta_1-\beta_0)(2m+x_{1s})+(1-\beta_0)(2m-x_{1l}))}{2\beta_1 m+1} - \frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right] & \text{if } 0 \leq x_1 \leq \frac{m}{m\beta_1+1}, \\ \frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0)) - \frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right] & \text{if } \frac{m}{m\beta_1+1} < x_1 \leq 1, \\ \frac{1}{4}(1-\beta_1)x_{1l}^2(2-x_{1l}), & \text{if } 1 < x_1 \leq 2. \end{cases}$$

3) $\mathbf{m = 2}$ && $\mathbf{\frac{1}{2}} \leq \beta_\mathbf{1} \leq \mathbf{\frac{3}{4}}$

In this case, $0 < 2(1-\beta_1)m-1 < \frac{m}{m\beta_1+1} < \frac{2m}{m+2} = 1$. The difference of this case with

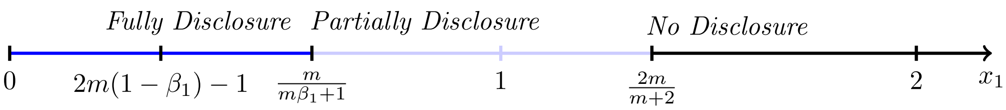


case 2) is when $x_1 \in [0, 2(1-\beta_1)m-1]$, the informed trader will disclose the information, and each follower will trade $x_2 = \frac{x_1+2}{m+1}$ in the second period, so the expected profit is $\frac{1}{4}x_1\frac{[(2-x_1+x_{1s})((\beta_1-\beta_0)(x_1+2)+(1-\beta_0)(2m-x_1))-(1-\beta_1)x_{1s}(2m-x_1)]}{(m+1)}(\theta_H-\theta_L)$. Hence, the expected profit is given as

$$\frac{E[\pi_1|\theta=\theta_H]}{\theta_H-\theta_L} = \begin{cases} \frac{1}{4}x_{1l}\frac{[(2-x_{1l}+x_{1s})((\beta_1-\beta_0)(x_{1l}+2)+(1-\beta_0)(2m-x_{1l}))-(1-\beta_1)x_{1s}(2m-x_{1l})]}{(m+1)} & \text{if } 0 \leq x_1 \leq 2m(1-\beta_1)-1, \\ \frac{1}{4}x_{1l}\left[\frac{(2-x_{1l}+x_{1s})((\beta_1-\beta_0)(2m+x_{1s})+(1-\beta_0)(2m-x_{1l}))}{2\beta_1 m+1} - \frac{x_{1s}(1-\beta_1)(2m-x_{1l})}{m+1}\right] & \text{if } 2m(1-\beta_1)-1 < x_1 \leq \frac{m}{m\beta_1+1}, \\ \frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0)) - \frac{x_{1s}(1-\beta_1)(2m-x_{1l})}{m+1}\right] & \text{if } \frac{m}{m\beta_1+1} < x_1 \leq 1, \\ \frac{1}{4}(1-\beta_1)x_{1l}^2(2-x_{1l}), & \text{if } 1 < x_1 \leq 2. \end{cases}$$

4) $\mathbf{m \geq 3}$ && $\mathbf{\frac{1}{2}} \leq \beta_\mathbf{1} \leq \mathbf{1} - \mathbf{\frac{1}{2m}}$

In this case, $0 < 2(1-\beta_1)m-1 < \frac{m}{m\beta_1+1} < 1 < \frac{2m}{m+2} < 2$. It is similar as case 3) except that there is one more interval $x_1 \in (1, \frac{2m}{m+1}]$. In this subdomain, the informed trader

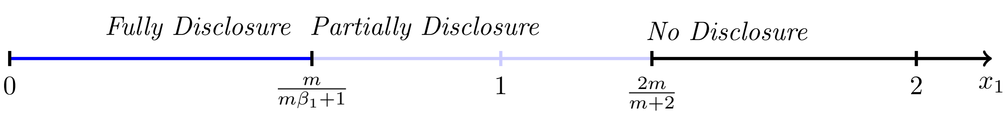


will disclose the information as long as $p_1 \neq \theta_H$ and each follower trades $x_2 = \frac{x_1+2}{m+1}$ in the second period, the expected profit is $\frac{1}{4}x_1\frac{(1-\beta_1)(2-x_1)(2m-x_1)}{(m+1)}(\theta_H-\theta_L)$. Thus, the expected profit is given as

$$\frac{E[\pi_1|\theta=\theta_H]}{\theta_H-\theta_L} = \begin{cases} \frac{1}{4}x_{1l}\frac{[(2-x_{1l}+x_{1s})((\beta_1-\beta_0)(x_{1l}+2)+(1-\beta_0)(2m-x_{1l}))-(1-\beta_1)x_{1s}(2m-x_{1l})]}{(m+1)} & \text{if } 0 \le x_1 \le 2m(1-\beta_1)-1, \\ \frac{1}{4}x_{1l}\left[\frac{(2-x_{1l}+x_{1s})((\beta_1-\beta_0)(2m+x_{1s})+(1-\beta_0)(2m-x_{1l}))}{2\beta_1 m+1} - \frac{x_{1s}(1-\beta_1)(2m-x_{1l})}{m+1}\right] & \text{if } 2m(1-\beta_1)-1 \le x_1 \le \frac{m}{m\beta_1+1}, \\ \frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0)) - \frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right] & \text{if } \frac{m}{m\beta_1+1} \le x_1 \le 1, \\ \frac{1}{4}x_1\frac{(1-\beta_1)(2-x_{1l})(2m-x_{1l})}{(m+1)}, & \text{if } 1 \le x_1 \le \frac{2m}{m+2}, \\ \frac{1}{4}(1-\beta_1)x_{1l}^2(2-x_{1l}), & \text{if } \frac{2m}{m+2} \le x_1 \le 2. \end{cases}$$

5) $\mathbf{m \geq 3}$ && $\mathbf{1 - \frac{1}{2m} \leq \beta_1 \leq 1}$

In this case, $2(1-\beta_1)m-1 < 0 < \frac{m}{m\beta_1+1} < 1 < \frac{2m}{m+2} < 2$. The only difference compared



with case 4) is that the interval $[0, 2(1-\beta_1)m-1]$ disappears since $2(1-\beta_1)m-1 < 0$, and the expected profit function is given as

$$\frac{E[\pi_1|\theta=\theta_H]}{\theta_H-\theta_L} = \begin{cases} \frac{1}{4}x_{1l}\left[\frac{(2-x_{1l}+x_{1s})((\beta_1-\beta_0)(2m+x_{1s})+(1-\beta_0)(2m-x_{1l}))}{2\beta_1 m+1} - \frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right] & \text{if } 0 \le x_1 \le \frac{m}{m\beta_1+1}, \\ \frac{1}{4}x_{1l}\left[(2-x_{1l}+x_{1s})((1-\beta_0)x_{1l}-x_{1s}(\beta_1-\beta_0)) - \frac{x_{1s}((1-\beta_1)(2m-x_{1l}))}{m+1}\right] & \text{if } \frac{m}{m\beta_1+1} \le x_1 \le 1, \\ \frac{1}{4}x_{1l}\frac{(1-\beta_1)(2-x_{1l})(2m-x_{1l})}{(m+1)}, & \text{if } 1 \le x_1 \le \frac{2m}{m+2}, \\ \frac{1}{4}(1-\beta_1)x_{1l}^2(2-x_{1l}), & \text{if } \frac{2m}{m+2} \le x_1 \le 2. \end{cases}$$

F The ex-post variance of fundamental value Σ_1 and Σ_2

We first compute Σ_1 in two cases: $0 \le x_1 < 1$ and $1 \le x_1 < 2$.

(1) When $0 \leq x_1 < 1$:

$$\begin{aligned}
\Sigma_1 &= E[Var(\theta|p_1)] \\
&= E[(\theta - p_1)^2] \\
&= E[(\theta_H - p_1)^2|\theta = \theta_H] * P(\theta = \theta_H) + E[(\theta_L - p_1)^2|\theta = \theta_L] * P(\theta = \theta_L) \\
&= \beta_0 E[(\theta_H - p_1)^2|\theta = \theta_H] + (1-\beta_0)E[(\theta_L - p_1)^2|\theta = \theta_L] \\
&= \beta_0 E[(\theta_H - p_M)^2|\theta = \theta_H, p_1 = p_M] * P(p_1 = p_M) \\
&\quad + \beta_0 E[(\theta_H - p_0)^2|\theta = \theta_H, p_1 = p_0] * P(p_1 = p_0) \\
&\quad + \beta_0 E[(\theta_H - \tilde{p}_M)^2|\theta = \theta_H, p_1 = \tilde{p}_M] * P(p_1 = \tilde{p}_M) \\
&\quad + (1-\beta_0)E[(\theta_L - \tilde{p}_M)^2|\theta = \theta_L, p_1 = \tilde{p}_M] * P(p_1 = \tilde{p}_M) \\
&\quad + (1-\beta_0)E[(\theta_L - p_0)^2|\theta = \theta_L, p_1 = p_0] * P(p_1 = p_0) \\
&\quad + (1-\beta_0)E[(\theta_L - p_M)^2|\theta = \theta_L, p_1 = p_M] * P(p_1 = p_M) \\
&= \beta_0 \frac{x_1}{2}(1-\beta_1)^2(\theta_H - \theta_L)^2 + \beta_0(1-x_1)(1-\beta_0)^2(\theta_H - \theta_L)^2 + \beta_0(1-\mu)\frac{x_1}{2}\beta_1^2(\theta_H - \theta_L)^2 \\
&\quad + (1-\beta_0)\frac{x_1}{2}(1-\beta_1)^2(\theta_H - \theta_L)^2 + (1-\beta_0)(1-x_1)(1-\beta_0)^2(\theta_H - \theta_L)^2 \\
&\quad + (1-\beta_0)(1-\mu)\frac{x_1}{2}\beta_1^2(\theta_H - \theta_L)^2 \\
&= \frac{1}{4}(1 - \frac{\mu}{2-\mu}x_1)(\theta_H - \theta_L)^2.
\end{aligned}$$

(2) When $1 \leq x_1 < 2$:

$$
\begin{aligned}
\Sigma_1 &= E[Var(\theta|p_1)] \\
&= E[(\theta - p_1)^2] \\
&= E[(\theta_H - p_1)^2|\theta = \theta_H] * P(\theta = \theta_H) + E[(\theta_L - p_1)^2|\theta = \theta_L] * P(\theta = \theta_L) \\
&= \beta_0 E[(\theta_H - p_1)^2|\theta = \theta_H] + (1 - \beta_0)E[(\theta_L - p_1)^2|\theta = \theta_L] \\
&= \beta_0 E[(\theta_H - p_M)^2|\theta = \theta_H, p_1 = p_M] * P(p_1 = p_M) \\
&\quad + \beta_0 E[(\theta_H - p_0)^2|\theta = \theta_H, p_1 = p_0] * P(p_1 = p_0) \\
&\quad + \beta_0 E[(\theta_H - \tilde{p}_M)^2|\theta = \theta_H, p_1 = \tilde{p}_M] * P(p_1 = \tilde{p}_M) \\
&\quad + (1 - \beta_0)E[(\theta_L - \tilde{p}_M)^2|\theta = \theta_L, p_1 = \tilde{p}_M] * P(p_1 = \tilde{p}_M) \\
&\quad + (1 - \beta_0)E[(\theta_L - p_0)^2|\theta = \theta_L, p_1 = p_0] * P(p_1 = p_0) \\
&\quad + (1 - \beta_0)E[(\theta_L - p_M)^2|\theta = \theta_L, p_1 = p_M] * P(p_1 = p_M) \\
&= \beta_0(1 - \frac{x_1}{2})(1 - \beta_1)^2(\theta_H - \theta_L)^2 + \beta_0(x_1 - 1)(1 - \mu)(1 - \beta_0)^2(\theta_H - \theta_L)^2 \\
&\quad + \beta_0(1 - \mu)(1 - \frac{x_1}{2})\beta_1^2(\theta_H - \theta_L)^2 + (1 - \beta_0)(1 - \frac{x_1}{2})(1 - \beta_1)^2(\theta_H - \theta_L)^2 \\
&\quad + (1 - \beta_0)(x_1 - 1)(1 - \mu)(1 - \beta_0)^2(\theta_H - \theta_L)^2 + (1 - \beta_0)(1 - \mu)(1 - \frac{x_1}{2})\beta_1^2(\theta_H - \theta_L)^2 \\
&= \frac{1}{4}\left[\frac{(1 - \mu)(2 + \mu)}{2 - \mu} - \frac{\mu(1 - \mu)}{2 - \mu}x_1\right](\theta_H - \theta_L)^2
\end{aligned}
$$

Then we compute Σ_2 under three cases: when there is no information disclosure and $0 \leq x_1 < 1$; when there is no information disclosure and $1 \leq x_1 < 2$; when there is information disclosure and $0 \leq x_1 < 1$ and $x_2 = \frac{x_1+2}{m+1}$.

When there is no information disclosure and $0 \leq x_1 < 1$:

$$\begin{aligned}
\Sigma_2 &= E[Var(\theta|p_2)] \\
&= E[(\theta - p_2)^2] \\
&= E[(\theta_H - p_2)^2|\theta = \theta_H] * P(\theta = \theta_H) + E[(\theta_L - p_2)^2|\theta = \theta_L] * P(\theta = \theta_L) \\
&= \beta_0 E[(\theta_H - p_2)^2|\theta = \theta_H] + (1-\beta_0)E[(\theta_L - p_2)^2|\theta = \theta_L] \\
&= \beta_0 E[(\theta_H - p_M)^2|\theta = \theta_H, p_2 = p_M] * P(p_2 = p_M) \\
&\quad +\beta_0 E[(\theta_H - p_0)^2|\theta = \theta_H, p_2 = p_0] * P(p_2 = p_0) \\
&\quad +\beta_0 E[(\theta_H - \tilde{p}_M)^2|\theta = \theta_H, p_2 = \tilde{p}_M] * P(p_2 = \tilde{p}_M) \\
&\quad +(1-\beta_0)E[(\theta_L - p_M)^2|\theta = \theta_L, p_2 = p_M] * P(p_2 = p_M) \\
&\quad +(1-\beta_0)E[(\theta_L - p_0)^2|\theta = \theta_L, p_2 = p_0] * P(p_2 = p_0) \\
&\quad +(1-\beta_0)E[(\theta_L - \tilde{p}_M)^2|\theta = \theta_L, p_2 = \tilde{p}_M] * P(p_2 = \tilde{p}_M) \\
&= \beta_0(1-\beta_1)^2[\frac{x_1}{2}(1-\frac{x_1}{2}) + (1-x_1)(1-\frac{x_1}{2}) - \mu(1-x_1)^2](\theta_H - \theta_L)^2 \\
&\quad +\beta_0(1-\beta_0)^2[(1-x_1)^2 + \frac{x_1^2}{4}(1-\mu)](\theta_H - \theta_L)^2 \\
&\quad +\beta_0\beta_1^2\frac{x_1}{2}(2-\frac{3}{2}x_1)(1-\mu)(\theta_H - \theta_L)^2 \\
&\quad +(1-\beta_0)(1-\beta_1)^2[\frac{x_1}{2}(1-\frac{x_1}{2}) + (1-x_1)(1-\frac{x_1}{2}) - \mu(1-x_1)^2](\theta_H - \theta_L)^2 \\
&\quad +(1-\beta_0)(1-\beta_0)^2[(1-x_1)^2 + \frac{x_1^2}{4}(1-\mu)](\theta_H - \theta_L)^2 \\
&\quad +(1-\beta_0)\beta_1^2\frac{x_1}{2}(2-\frac{3}{2}x_1)(1-\mu)(\theta_H - \theta_L)^2 \\
&= \frac{1}{4}[(1-\beta_1)^2(2-x_1)^2 + (1-4\mu(1-\beta_1)^2)(1-x_1)^2 + \frac{1-\mu}{4}x_1 + \beta_1^2(1-\mu)(4x_1 - 3x_1^2)](\theta_H - \theta_L)^2.
\end{aligned}$$

When there is no information disclosure and $1 \leq x_1 < 2$:

$$
\begin{aligned}
\Sigma_2 &= E[Var(\theta|p_2)] \\
&= E[(\theta - p_2)^2] \\
&= E[(\theta_H - p_2)^2|\theta = \theta_H] * P(\theta = \theta_H) + E[(\theta_L - p_2)^2|\theta = \theta_L] * P(\theta = \theta_L) \\
&= \beta_0 E[(\theta_H - p_2)^2|\theta = \theta_H] + (1 - \beta_0)E[(\theta_L - p_2)^2|\theta = \theta_L] \\
&= \beta_0 E[(\theta_H - p_M)^2|\theta = \theta_H, p_2 = p_M] * P(p_2 = p_M) \\
&\quad + \beta_0 E[(\theta_H - p_0)^2|\theta = \theta_H, p_2 = p_0] * P(p_2 = p_0) \\
&\quad + \beta_0 E[(\theta_H - \tilde{p}_M)^2|\theta = \theta_H, p_2 = \tilde{p}_M] * P(p_2 = \tilde{p}_M) \\
&\quad + (1 - \beta_0)E[(\theta_L - p_M)^2|\theta = \theta_L, p_2 = p_M] * P(p_2 = p_M) \\
&\quad + (1 - \beta_0)E[(\theta_L - p_0)^2|\theta = \theta_L, p_2 = p_0] * P(p_2 = p_0) \\
&\quad + (1 - \beta_0)E[(\theta_L - \tilde{p}_M)^2|\theta = \theta_L, p_2 = \tilde{p}_M] * P(p_2 = \tilde{p}_M) \\
&= \beta_0(1 - \beta_1)^2[\mu(1 - \frac{x_1}{2})^2 + (1 - \mu)(1 - \frac{x_1}{2})^2](\theta_H - \theta_L)^2 \\
&\quad + \beta_0(1 - \beta_0)^2[(1 - \mu)(x_1 - 1) + (1 - \mu)(1 - \frac{x_1}{2})^2](\theta_H - \theta_L)^2 \\
&\quad + \beta_0\beta_1^2(1 - \mu)(1 - \frac{x_1}{2})^2(\theta_H - \theta_L)^2 \\
&\quad + (1 - \beta_0)(1 - \beta_1)^2[\mu(1 - \frac{x_1}{2})^2 + (1 - \mu)(1 - \frac{x_1}{2})^2](\theta_H - \theta_L)^2 \\
&\quad + (1 - \beta_0)(1 - \beta_0)^2[(1 - \mu)(x_1 - 1) + (1 - \mu)(1 - \frac{x_1}{2})^2](\theta_H - \theta_L)^2 \\
&\quad + (1 - \beta_0)\beta_1^2(1 - \mu)(1 - \frac{x_1}{2})^2(\theta_H - \theta_L)^2 \\
&= \frac{1}{4}[(1 - \beta_1)(2 - x_1)^2 + \frac{1 - \mu}{4}x_1](\theta_H - \theta_L)^2.
\end{aligned}
$$

When there is information disclosure and $0 \leq x_1 < 1$ and $x_2 = \frac{x_1+2}{m+1}$:

$$\begin{aligned}
\Sigma_2 &= E[Var(\theta|p_2)] \\
&= E[(\theta - p_2)^2] \\
&= E[(\theta_H - p_2)^2|\theta = \theta_H] * P(\theta = \theta_H) + E[(\theta_L - p_2)^2|\theta = \theta_L] * P(\theta = \theta_L) \\
&= \beta_0 E[(\theta_H - p_2)^2|\theta = \theta_H] + (1 - \beta_0)E[(\theta_L - p_2)^2|\theta = \theta_L] \\
&= \beta_0 E[(\theta_H - p_M)^2|\theta = \theta_H, p_2 = p_M] * P(p_2 = p_M) \\
&\quad +\beta_0 E[(\theta_H - p_0)^2|\theta = \theta_H, p_2 = p_0] * P(p_2 = p_0) \\
&\quad +\beta_0 E[(\theta_H - \tilde{p}_M)^2|\theta = \theta_H, p_2 = \tilde{p}_M] * P(p_2 = \tilde{p}_M) \\
&\quad +(1 - \beta_0)E[(\theta_L - p_M)^2|\theta = \theta_L, p_2 = p_M] * P(p_2 = p_M) \\
&\quad +(1 - \beta_0)E[(\theta_L - p_0)^2|\theta = \theta_L, p_2 = p_0] * P(p_2 = p_0) \\
&\quad +(1 - \beta_0)E[(\theta_L - \tilde{p}_M)^2|\theta = \theta_L, p_2 = \tilde{p}_M] * P(p_2 = \tilde{p}_M) \\
&= \frac{1}{4}\{\mu(1 - \beta_1)^2(2 - \frac{2m - x_1}{m + 1})(2 - x_1) + (1 - \mu)[(1 - x_1)(\frac{2m - x_1}{m + 1} - 1) + \frac{x_1}{4}\frac{2m - x_1}{m + 1}] \\
&\quad +(1 - \mu)\beta_1^2 x_1(1 - \frac{2m - x_1}{m + 1})\}(\theta_H - \theta_L)^2.
\end{aligned}$$

References

Abreu, D. and M. K. Brunnermeier (2002). Synchronization risk and delayed arbitrage. *Journal of Financial Economics 66*(2), 341–360.

Abreu, D. and M. K. Brunnermeier (2003). Bubbles and crashes. *Econometrica 71*(1), 173–204.

Admati, A. R. and P. Pfleiderer (1986). A monopolistic market for information. *Journal of Economic Theory*.

Admati, A. R. and P. Pfleiderer (1988). Selling and trading on information in financial markets. *The American Economic Review*.

Bommel, J. V. (2003). Rumors. *The Journal of Finance 58*(4), 1499–1520.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990). Noise trader risk in financial markets. *Journal of Political Economy 98*(4), 703–738.

Fishman, M. J. and K. M. Hagerty (1995). The incentive to sell financial market information. *Journal of Financial Intermediation*.

Foster, F. and S. Viswanathan (1996). Strategic trading when agents forecast the forecasts of others. *Journal of Finance 51*(4), 1437–78.

Gromb, D. and D. Vayanos (2010, March). Limits of arbitrage: The state of the theory. Working Paper 15821, National Bureau of Economic Research.

Grossman, S. and J. Stiglitz (1982). On the impossibility of informationally efficient markets: reply. *American Economic Review*.

Kovbasyuk, S. and M. Pagano (2015). Advertising arbitrage. *Available at SSRN: https://ssrn.com/abstract=2509735*.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*, 1315–1335.

Ljungqvist, A. and W. Qian (2016). How constraining are limits to arbitrage? *The Review of Financial Studies 29*(8), 1975.

Schmidt, D. (2017). Stock Market Rumors and Credibility. *Working Paper*.

Shleifer, A. and R. W. Vishny (1997). The limits of arbitrage. *The Journal of Finance*.

Stein, J. C. (2008, November). Conversations among Competitors. *The American Economic Review 98*(5), 2150–2162.

Steven Huddart, John S. Hughes, C. B. L. (2001). Public disclosure and dissimulation of insider trades. *Econometrica 69*(3), 665–681.

Swem, N. (2017). Information in financial markets: Who gets it first? *Available at SSRN: https://ssrn.com/abstract=2437733*.

Yang, L. and H. Zhu (2017). Back-running: Seeking and hiding fundamental information in order flows. *Rotman School of Management Working Paper No. 2583915. Available at SSRN: https://ssrn.com/abstract=2583915*.